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Quality Quandaries: Interpretation of Signals from Runs Rules in Shewhart Control Charts

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INTRODUCTION

The Shewhart control chart is for many people the embodiment of statistical process control (SPC). In the past, many modifications and improvements have been devised (see Woodall and Montgomery [1999] for a brief overview). Several modifications or additions were aimed at more power for out-of-control situations. Among the earlier additions belong the runs rules from the Western Electric Company (1956). These rules stem from the idea that a graphical pattern in the chart may help in identifying an out-of-control situation even before the control limits have been exceeded. Runs rules are formalizations of such patterns and therefore easily understood by users. Although many rules have been devised, for all kinds of patterns, only a few should be selected in practice, because application of many rules simultaneously leads to an unacceptable number of false signals (Does and Schriever 1992), and users might easily get confused.

Many alternatives for the Shewhart control chart have been designed with an increased power for specific out-of-control situations in mind (see Woodall and Montgomery [1999] for references). For example, a small change of the process mean will generally be detected faster with a cumulative sum (CUSUM) or an exponentially weighted moving average (EWMA) chart. Many processes, however, will get out of control not just for one reason but because of one of many. Each failure cause usually has its own specific effect on the process data: the mean may change more or less, or the variation, or both. SPC is aimed at finding and correcting permanent changes as quickly as possible, whereas feedback control is generally a better alternative for correcting gradual or temporary changes. Usually, there is no single optimal control chart for all possible failure causes. The Shewhart chart with additional runs rules might be considered as an omnibus test for many different permanent process disturbances.

An effective SPC system requires an out-of-control action plan (OCAP; see Sandorf and Bassett 1993) to accompany a control chart. An OCAP is a guide for the user to solve the problem when an out-of-control signal is given. The form may range from a simple checklist to an extensive flowchart

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with questions and answers. OCAPs aim at quick and uniform solutions to problems and therefore reflect the available knowledge about the process. An effective OCAP demands that most possibilities of failure are known, as well as solutions. Permanent fixes are preferred, but if a temporary fix is the only possibility, then it should be part of the OCAP. The order of checking the different failure causes is often important for a speedy recovery of a problem. We will demonstrate that a signal in a control chart with additional runs rules may also provide additional information about the most likely out-of-control failure causes.

FRAMEWORK

A nearly exhaustive list of out-of-control causes and effects is the starting point. In an ideal process all causes would be permanently fixed, but in the real world this is impossible or too expensive for most causes. These causes are therefore part of the OCAP. Knowledge of the (relative) frequencies of the causes may help to find the problem quickly. Causes with serious consequences might get priority, however, or the ones that are easy to check, even if they have low frequency. A cause-and-effect analysis—giving this information—is an important element of a systematic approach for introducing SPC (Does et al. 1999).

Assume that the failure causes C_1, C_2, ..., C_n are included in the OCAP. The relative frequencies of these causes will approximately be known from the history of the process. This information is needed, because chances are higher (in general) that a problem is solved quickly if the reason is first sought among the most frequently occurring causes. The relative frequency of cause C_i is quantified as f_i ($i = 1, \ldots, n$). In a cause-and-effect analysis the frequencies are rated from 1 to 10 (or 1 to 5), but in fact any rating with higher numbers for higher frequencies will do.

Assume that decision rules R_1, R_2, ..., R_m are used in a control chart with additional runs rules. If an out-of-control signal is given, then at least one of these rules is responsible. There is a probability of a false signal, but the OCAP will be activated anyway, to find the failure cause. In a stable system, with failure causes having reproducible effects, probability relations with the decision rules emerge if the system would be observed long enough. We use these relations to make more efficient OCAPs. When a decision rule is responsible for a signal, the failure causes with the highest conditional probabilities of occurring, given this rule, are the most likely candidates. We therefore need to establish the conditional probabilities $P(C_i|R_j)$ for all $i \in \{1, \ldots, n\}$ and $j \in \{1, \ldots, m\}$.

$$\mathrm{P}(\mathrm{C_i}|\mathrm{R_j}) = \frac{\mathrm{P}(\mathrm{C_i R_j})}{\mathrm{P}(\mathrm{R_j})} = \frac{\mathrm{P}(\mathrm{R_j}|\mathrm{C_i}) * \mathrm{P}(\mathrm{C_i})}{\mathrm{P}(\mathrm{R_j})} \propto \mathrm{P}(\mathrm{R_j}|\mathrm{C_i}) * \mathrm{f_i} \quad j \in 1, \ldots, m \text{ fixed} \qquad [1]$$

For these conditional probabilities we need to know the conditional probabilities $P(R_j|C_i)$. Note that this is in fact a rather Bayesian approach; the prior frequencies f_i of the failure causes C_i are improved by the posterior probabilities $P(C_i|R_j)$.

Estimating the conditional probabilities $P(R|C)$ from observed frequencies is virtually impossible, because an in-control process has by definition only few signals. But the probabilities can be inferred from the supposedly known effects of failure causes and the probability distribution of the process.

For example, if R_1 is the standard Shewhart decision rule (based on three-sigma control limits), and C_1 is a failure cause with the effect that the process standard deviation gets twice as large (without changing the process mean), then $P(R_1|C_1) = 0.1336$ (assuming that the in-control process has a normal distribution with independent observations). And if the effect of failure cause C_2 is a shift of the process mean amounting to 0.5σ (with the same standard deviation), then $P(R_1|C_2) = 0.00644$. These examples show that the quantitative effects of all failure causes on the parameters of the distribution should be known. Educated guesses based on the qualitative descriptions that people normally give are in many cases the best one can achieve. But analysis of past data, or observed effects from designed experiments, can sometimes be used for better estimates.

Now assume that decision rule R_2 is this runs rule: a signal is given when two out of three consecutive data points are beyond one of the two-sigma limits ("warning limits"). The rule R_2 is used as an addition to rule R_1, so there will be more signals than with R_1 only. In this article only the additional signals are attributed to rule R_2; all other signals are attributed to rule R_1 even if R_2 would signal as well. It is common practice to attribute signals to the shortest (least complicated) rule (see Göb et al. [2001] for another example). Calculating $P(R_2|C_1)$ is therefore not as simple as calculating $P(R_1|C_1)$. Another complication is that $P(R_2|C_1)$ depends on the number of observations since the failure cause. The probability is obviously 0 at the first observation. However, after some observations the probability stabilizes. A solution for calculating the *average* probability $P(R_2|C_1)$ can be obtained by regarding the process as a Markov chain. Brook and Evans (1972) used the Markov chain approach to derive the probability distribution of the run length of CUSUM charts. Champ and Woodall (1987) applied the method to a Shewhart chart with supplementing runs rules. The above example with R_1 and R_2 is described in more detail in Champ and Woodall (1990). For our purpose (i.e., the calculation of the average probabilities $P(R|C)$), we do not need the distribution of the run lengths. It is sufficient to know the average run length (ARL). For example, with failure cause C_1 the average run length of the combined rules R_1 and R_2 is 6.279. The probability of a signal is therefore 0.1593. Because $P(R_1|C_1) = 0.1336$, it follows that $P(R_2|C_1) = 0.0256$.

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DECISION RULES

The Western Electric Company (1956) defines eight runs rules. Nelson (1984) recommended the same eight rules. A further reduction is obligatory in a manually operated system, and we decided to limit ourselves to only four really simple decision rules:

Rule R_1: one value beyond one of the three-sigma limits.
Rule R_2: two out of three consecutive values beyond one of the two-sigma limits.
Rule R_3: four out of five consecutive values beyond one of the one-sigma limits.
Rule R_4: nine consecutive values above or below the central line.

This combination of rules is selected by other authors as well (Alwan et al. 1994; Fu et al. 2003; Göb et al. 2001), although they use eight consecutive points in rule R_4. This system of decision rules can be treated as a Markov chain and is therefore suited for the computational methods of the previous section.

We deliberately skipped the familiar rule with six consecutive increasing or decreasing values, especially because the power is small (Trip and Wieringa 2006; or in a different context, Aparisi et al. 2004). Another disadvantage of this rule is that it does not fit into a Markov (finite) chain framework. Monte Carlo simulations would therefore be needed to investigate the complete system, leading to approximate results only.

With the method of the previous section, the probabilities $P(R_i|C)$ $(i = 1, \ldots, 4)$ can be calculated for any failure cause C with known effects on the process mean and standard deviation. But, due to interfering rules, this method will slightly underestimate the probabilities of the longer rules, as Figure 1 illustrates.

An SPC system with rules R_1–R_4 gives two signals, at observations 8 (R_3) and 10 (R_2). Without R_3 there would be a single signal at observation 9, and with just R_4 there would be a single signal at observation 12. The example shows that a shorter rule may give way to a longer rule. It also shows that the longer rule may be "interrupted" by a shorter one. The latter case will obviously occur more often. That is why the ARLs of R_1, $R_1 + R_2$, $R_1 + R_2 + R_3$, and $R_1 + R_2 + R_3 + R_4$ are used to compute the separate contributions from R_1, R_2, R_3, and R_4, respectively. The contribution of R_2 will therefore slightly be overestimated at the expense of R_3, and R_4.

Exact results are possible, but the price to pay is to distinguish between all possible absorption states (Champ and Woodall 1997). In the system with four decision rules there are 15 different absorption states

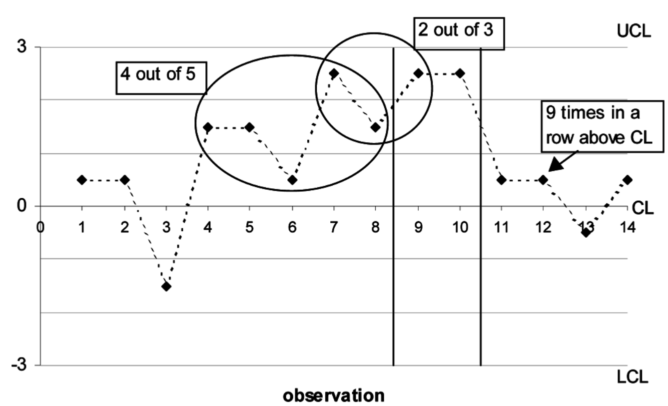

FIGURE 1 Example of interfering runs rules.

(or 30 in case of a nonsymmetrical situation). The number of different OCAP schemes would be the same and therefore hardly useful in daily practice.

The reader will have noticed the implicit and obvious assumption that the process continues with a clean slate after a signal is given. After all, SPC is a tool for adjusting or repairing a process to bring it in control again.

TABLES OF CONTRIBUTIONS OF SEPARATE RUNS RULES

This section presents tables for a Shewhart chart with the decision rules of the previous section. For the in-control process we assume a standard normal distribution with independent observations, and the control limits of the Shewhart chart are ±3. The discussion is aimed at samples of size 1 ("individual measurements"), but the results can also be applied to a chart of the sample means. For sample sizes larger than 1 a separate chart for the sample variation (e.g., an R-chart) is much more efficient to monitor the process variation. For individual measurements the moving range (MR) chart, which might be considered as an alternative for the R-chart for this particular case, is not a good alternative (Trip and Wieringa 2006).

The average run lengths of the in-control process are given in Table 1.

The ARL in an SPC system with the four decision rules is 109.0479, so the probability of a false signal is 0.009170. Rules R_1–R_4 account separately for probabilities 0.002700, 0.001736, 0.003089, and 0.001645, respectively. The contributions of R_1–R_4 to the total number of signals are 29.4, 18.9, 33.7, and 17.9%. These percentages are used for comparisons with out-of-control situations in Tables 2a–2d.

TABLE 1 Average Run Lengths of the In-Control Process

Rule (s)	ARL
1	370.3794
1+2	225.4325
1+3	166.0509
1+4	216.6891
1+2+3	132.8908
1+2+4	158.7345
1+3+4	130.1834
1+2+3+4	109.0479

TABLE 2a Contribution of Decision Rule R_1

		μ							
R_1		0	0.25	0.5	1	1.5	2	2.5	3
	0.5	0.0	0.0	0.0	0.0	0.6	7.4	**33.9**	**76.1**
	0.75	2.6	1.7	1.7	3.8	11.6	29.2	**55.2**	**79.7**
	0.9	17.1	12.4	10.0	13.2	24.1	**42.2**	**63.6**	**81.9**
	1	***29.4***	23.9	19.7	22.1	**33.1**	**49.7**	**68.0**	**83.3**
σ	1.1	**39.7**	**35.2**	**30.4**	**31.6**	**41.6**	**56.3**	**71.8**	**84.6**
	1.25	**51.3**	**48.7**	**45.0**	**45.0**	**52.9**	**64.5**	**76.4**	**86.3**
	1.5	**64.6**	**63.8**	**62.3**	**62.3**	**67.1**	**74.5**	**82.2**	**88.8**
	2	**79.9**	**79.9**	**79.9**	**80.7**	**82.8**	**85.8**	**89.1**	**92.3**
	2.5	**87.7**	**87.8**	**87.9**	**88.6**	**89.8**	**91.2**	**92.9**	**94.5**
	3	**92.0**	**92.0**	**92.1**	**92.6**	**93.3**	**94.1**	**95.0**	**96.0**

Reference contribution in bold italic; contributions exceeding the reference contribution in bold. Values in percentages.

TABLE 2b Contribution of Decision Rule R_2

		μ							
R_2		0	0.25	0.5	1	1.5	2	2.5	3
	0.5	0.0	0.0	0.0	0.8	16.9	**64.5**	**64.5**	**23.8**
	0.75	2.3	2.5	3.6	12.4	**33.5**	**49.8**	**41.4**	**20.1**
	0.9	12.5	12.0	13.2	**22.1**	**35.6**	**41.2**	**32.6**	17.6
	1	***18.9***	18.7	**19.7**	**26.4**	**34.7**	**36.2**	**28.2**	16.1
σ	1.1	**22.6**	**22.8**	**23.8**	**28.3**	**32.8**	**31.9**	**24.6**	14.7
	1.25	**24.1**	**24.6**	**25.6**	**28.0**	**29.0**	**26.5**	**20.3**	12.9
	1.5	**22.2**	**22.4**	**23.0**	**23.6**	**22.7**	**19.7**	15.3	10.4
	2	15.3	15.3	15.2	14.7	13.6	11.7	9.4	7.1
	2.5	10.2	10.2	10.1	9.5	8.7	7.6	6.3	5.0
	3	7.0	7.0	6.9	6.5	6.0	5.3	4.5	3.7

Reference contribution in bold italic; contributions exceeding the reference contribution in bold. Values in percentages.

TABLE 2c Contribution of Decision Rule R_3

		μ							
R_3		0	0.25	0.5	1	1.5	2	2.5	3
	0.5	0.1	0.6	4.0	**54.1**	**80.9**	28.1	1.6	0.0
	0.75	17.8	22.3	31.4	**55.4**	**51.2**	21.0	3.4	0.2
	0.9	**34.7**	**37.6**	**41.9**	**47.4**	**37.0**	16.5	3.8	0.4
	1	***33.7***	**36.7**	**48.9**	**40.1**	29.6	13.9	3.8	0.5
σ	1.1	28.6	**31.2**	**34.0**	32.7	23.6	11.6	3.6	0.7
	1.25	20.9	22.4	24.4	23.3	16.8	8.9	3.2	0.8
	1.5	12.3	12.7	13.4	13.0	9.7	5.7	2.5	0.8
	2	4.7	4.7	4.7	4.4	3.6	2.5	1.4	0.7
	2.5	2.0	2.0	2.0	1.8	1.5	0.5	0.8	0.5
	3	1.0	1.0	0.9	0.9	0.8	0.6	0.4	0.3

Reference contribution in bold italic; contributions exceeding the reference contribution in bold. Values in percentages.

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TABLE 2d **Contribution of Decision Rule R_4**

R_4		μ							
		0	0.25	0.5	1	1.5	2	2.5	3
	0.5	**99.9**	**99.4**	**96.0**	**45.1**	1.5	0.0	0.0	0.0
	0.75	**77.3**	**73.5**	**63.3**	**28.4**	3.7	0.1	0.0	0.0
	0.9	**35.7**	**38.0**	**34.9**	17.2	3.3	0.1	0.0	0.0
	1	***17.9***	**20.7**	11.7	11.4	2.6	0.2	0.0	0.0
σ	1.1	9.1	10.8	11.8	7.3	2.0	0.2	0.0	0.0
	1.25	3.6	4.3	5.0	3.6	1.2	0.2	0.0	0.0
	1.5	1.0	1.1	1.3	1.1	0.5	0.1	0.0	0.0
	2	0.1	0.1	0.1	0.1	0.1	0.0	0.0	0.0
	2.5	0.0	0.0	0.0	0.0	0.0	0.7	0.0	0.0
	3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Reference contribution in bold italic; contributions exceeding the reference contribution in bold. Values in percentages.

A persistent shift of the mean and/or simultaneous change of the standard deviation will be the result of a failure cause. For shifts from 0 to 3, and for standard deviations from 0.5 to 3, ARLs are calculated. Tables 2a–2d give the relative contributions of the different decision rules to the total number of signals for several out-of-control processes. Contributions exceeding the reference percentages (i.e., the cell with $\mu = 0$ and $\sigma = 1$; corresponding percentage is indicated in bold italic in the tables) are printed in boldface, meaning that the decision rule is more sensitive than in an in-control process. The decision rule with the largest excess is shown in Table 3.

R_1 is in particular sensitive to large changes of the mean and/or the standard deviation. R_4, on the other hand, is especially sensitive to small changes of the mean with less than usual variation. R_2 and R_3 are somewhere between these extremes, with R_2 more sensitive to larger deviations than R_3.

TABLE 3 **The Decision Rule with the Largest Excess Over the Reference Contribution**

		μ							
		0	0.25	0.5	1	1.5	2	2.5	3
	0.5	4	4	4	4	3	2	2	1
	0.75	4	4	4	3	2	2	1	1
	0.9	4	4	4	3	2	2	1	1
σ	1	ref.	3	3	2	2	1	1	1
	1.1	1	1	2	2	2	1	1	1
	1.25	1	1	1	1	1	1	1	1
	1.5	1	1	1	1	1	1	1	1
	2	1	1	1	1	1	1	1	1

Tables 2a–2d indicate that the contributions of the decision rules are sensitive to parameters μ and σ. The consequence is that the estimates of the effects of a failure cause on the process are important for the designed OCAP. Getting good estimates is one of the toughest problems, however. It is necessary to learn from the process while using the OCAP and to update the method whenever new substantial information is available.

CASE STUDY

Douwe Egberts, a Sara Lee International subsidiary, is the leading manufacturer and distributor of coffee and tea in The Netherlands. In one of its facilities, instant coffee and liquid coffee extract are produced. One of the key processes is the extraction of coffee from roasted beans, taking place in a series of cylinders and connected with storage tanks. The time it takes to extract coffee from one cylinder is the cycle time of the process. The cycle time is a good indicator whether or not the process runs smoothly and is therefore selected for monitoring with SPC methods. Many failure causes will show up in longer cycle times than usual. The in-control cycle time is on average 30 minutes and 13 seconds with a standard deviation of 1 minute and 39 seconds.

Along the lines of Does et al. (1999), a team was set up for introducing a control chart and OCAP for the cycle time. The team identified 12 important failure causes with consequences for the cycle time, amounting to about 90% of all failures:

- C_1–C_4: leakage of pumps or tubes (at four spots);
- C_5: defects in the control system of the equipment;
- C_6–C_9: contamination of the tubes or filters (at four spots);
- C_{10}: damaged grinding mill;
- C_{11}–C_{12}: broken stirring devices (at two spots).

The frequencies of these causes were established from the experience of operators and from the knowledge and database of maintenance technicians. The effects of the failure causes on cycle time could partly be determined from an extended database with detailed process information. As an example, the failure cause "contamination of the filter before the tap tank" is explained here. On a scale from 1 to 10 ("hardly ever" to "nearly always") the frequency of

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TABLE 4 Properties of 12 Failure Causes

Failure cause	Frequency	Effect on: μ	σ	Conditional probabilities (%) R_1	R_2	R_3	R_4	Scores $\propto P(C_i\|R_j)$ R_1	R_2	R_3	R_4
C_1	3	2	0.5	7.4	64.5	28.1	0.0	22.2	193.5	84.3	0.0
C_2	3	0.2	1	25.0	18.7	36.1	20.1	75.0	56.1	108.3	60.3
C_3	5	0.5	1	19.7	19.7	48.9	11.7	98.5	98.5	244.5	58.5
C_4	5	0.5	1.5	62.3	23.0	13.4	1.3	311.5	115.0	67.0	6.5
C_5	2	0	2	79.9	15.3	4.7	0.1	159.8	30.6	9.4	0.2
C_6	5	0.3	1.1	34.2	23.0	31.8	11.0	171.0	115.0	159.0	55.0
C_7	8	0.3	1.1	34.2	23.0	31.8	11.0	273.6	184.0	254.4	88.0
C_8	8	0.1	1.25	50.3	24.3	21.5	3.9	402.4	194.4	172.0	31.2
C_9	3	1.2	0.75	6.9	20.8	53.7	18.5	20.7	62.4	161.1	55.5
C_{10}	3	3	1	83.3	16.1	0.5	0.0	249.9	48.3	1.5	0.0
C_{11}	4	0	2	79.9	15.3	4.7	0.1	319.6	61.2	18.8	0.4
C_{12}	9	0	1.25	51.3	24.1	20.9	3.6	461.7	216.9	188.1	32.4

this cause was given the value 8. The effect is that the pressure in the extraction cylinder rises and that the process becomes somewhat less stable. The consequence on the cycle time will be that both the standard deviation and the average increase "a bit." Pressing the team for numbers they estimated the effect as 10% more standard deviation and 30-second longer cycle times on average. Of course these were only rough estimates, but a sample from the database pointed in the same direction. More reliable data required more investigation, but the weak point was that past failure causes have not been recorded adequately. The historical database was therefore of limited use for this investigation (as is often the case with historical databases), but for a start the information was sufficient.

A 30-second increase of the cycle time is 0.3 standard deviations. From Tables 2a–2d we can infer the conditional probabilities of the decision rules given this failure cause from the entries at $\mu = 0.25$ and $\sigma = 1.1$ (interpolation between the entries at $\mu = 0.25$ and $\mu = 0.5$ would give slightly better results). The probabilities of a signal would be 35.2, 22.8, 31.2, and 10.8% for rules 1–4 respectively. Table 4 contains the conditional probabilities of a signal of the four decision rules for the 12 failure causes of the SPC team.

For entries of μ not in Tables 2a–2d, probabilities were obtained through interpolation. The last columns of Table 4 are calculated with formula [1], to give the "scores," which are proportional to the conditional probabilities $P(C_i|R_j)$. They show that the most likely failure cause depends on the rule that issued a signal. A signal from R_1 or R_2 indicates that C_{12} is the most likely failure cause, but a signal from R_3 or R_4 points at C_7 as the most likely candidate. The complete order of likely failure causes for the different decision rules is given in Table 5.

Some failure causes are difficult to detect; for example, it can be hard to pinpoint the exact location of a leakage. The use of Table 5 is that inspection can be more to the point, and searching time is reduced. For example, C_1 is generally the least probable among the four failure causes related to leakage (C_1–C_4) but it is suddenly the most likely one if R_2 is signaling. Thanks to this analysis, a much more advanced scheme for searching failure causes was introduced than originally intended. The OCAP was only an intermediate solution, however, because most failure causes were permanently eliminated one after another.

TABLE 5 Most Likely Failure Causes

	Order of most likely failure causes: 1	2	3	4	5	6	7	8	9	10	11	12
R_1	C_{12}	C_8	C_{11}	C_4	C_7	C_{10}	C_6	C_5	C_3	C_2	C_1	C_9
R_2	C_{12}	C_8	C_1	C_7	C_4/C_6		C_3	C_9	C_{11}	C_2	C_{10}	C_5
R_3	C_7	C_3	C_{12}	C_8	C_9	C_6	C_2	C_1	C_4	C_{11}	C_5	C_{10}
R_4	C_7	C_2	C_3	C_9	C_6	C_{12}	C_8	C_4	C_{11}	C_5	C_1/C_{10}	

SUMMARY AND CONCLUSIONS

Runs rules are natural extensions to the standard Shewhart control chart. This article shows that additional advantage of runs rules can be achieved

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for problem solving, because the particular decision rule that gives an out-of-control signal is also informative about the state of the current process. Because the effects of different failure causes on the process are also different, the two can be combined in order to speed up problem solving. A case study illustrates potential benefits.

A successful implementation of the method requires, on the one hand, that failure causes and effects are summarized quantitatively. This is partly an extension of the well-structured method of Does et al. (1999) for the introduction of SPC. The most difficult part is to quantify the effects of failure causes on the process; historical databases and designed experiments might be used for improving subjective estimates. Experience in time will also help to improve estimates. A regular update of Table 4, leading to a dynamic OCAP, will therefore be necessary. The other requirement is to know how a Shewhart control chart with additional runs rules performs in a state of control, and under all kinds of deviations. The article shows that nearly exact results can be obtained for a set of commonly used runs rules, for which the process can be regarded as a Markov chain.

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