Another Look at the EWMA Control Chart with Estimated Parameters

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When in-control process parameters are estimated, Phase II control chart performance will vary among practitioners due to the use of different Phase I data sets. The typical measure of Phase II control chart performance, the average run length (ARL), becomes a random variable due to the selection of a Phase I data set for estimation. Aspects of the ARL distribution, such as the standard deviation of the average run length (SDARL), can be used to quantify the between-practitioner variability in control chart performance. In this article, we assess the in-control performance of the exponentially weighted moving average (EWMA) control chart in terms of the SDARL and percentiles of the ARL distribution when the process parameters are estimated. Our results show that the EWMA chart requires a much larger amount of Phase I data than previously recommended in the literature in order to sufficiently reduce the variation in the chart performance. We show that larger values of the EWMA smoothing constant result in higher levels of variability in the in-control ARL distribution; thus, more Phase I data are required for charts with larger smoothing constants. Because it could be extremely difficult to lower the variation in the in-control ARL values sufficiently due to practical limitations on the amount of the Phase I data, we recommend an alternative design criterion and a procedure based on the bootstrap approach.

Key Words: Bootstrap; Estimation Effect; SDARL; SPC; Standard Deviation of Average Run Length; Statistical Process Control.

1. Introduction

The exponentially weighted moving average (EWMA) control chart was first introduced by Roberts (1959). The EWMA chart statistic is a weighted average of measurements, giving heaviest weights to the most recent observations. This provides the chart with the advantage of being sensitive to small- and moderate-sized sustained shifts in the

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process parameters. As a consequence, the EWMA chart is one of the primary alternatives to Shewhart control charts when small shifts in the parameters are to be detected quickly. The EWMA chart has received a great deal of attention in the statistical process control (SPC) literature. See, for example, Crowder (1987, 1989), Robinson and Ho (1978), Lucas and Saccucci (1990), Steiner (1999), Jones et al. (2001), Jones (2002), Simões et al. (2010), and Zwetsloot et al. (2014, 2015).

Phase II control charts are designed for monitoring processes and detecting deviations from the in-control values of the process parameter(s). Because the true values of the in-control parameters are rarely known in practice, practitioners typically begin by collecting baseline information on the process. Practitioners gather m samples each of size $n \geq 1$ that constitute the Phase I data set. The Phase I data are used to evaluate the stability of the process and determine an in-control reference sample from which estimates of the process parameters can be obtained. These parameter estimates are then used to design a suitable Phase II control chart, with the aim of quickly detecting out-of-control conditions. For overviews of Phase I methods, see Chakraborti et al. (2009) and Jones-Farmer et al. (2014).

When we refer to the "amount of Phase I data" in our paper, we refer to the total number of observations in Phase I, i.e., mn. We do not consider applications in which there are two or more components of variation in the data, although these occur frequently in practice and deserve study. The use of control charts when there are several components of variation was discussed by Woodall and Thomas (1995).

The performance of control charts with estimated parameters has received a great deal of attention in the SPC literature. See, for example, Quesenberry (1993), Chen (1997), and Jones et al. (2001, 2004). Jensen et al. (2006) and Psarakis et al. (2014) provided reviews of the literature on the performance of control charts with estimated parameters. The general consensus is that the use of parameter estimates results in control charts with less predictable statistical performance than those with known parameters.

Phase II control chart performance is commonly evaluated using characteristics of the run length distribution. The run length of a control chart is a random variable defined as the number of the plotted statistics until the chart signals. One of the most common measures of Phase II control chart performance is the average run length (ARL). When parameters are estimated the control chart performance will depend on the estimated parameters and will thus vary among practitioners. This is because practitioners use different Phase I data sets, which result in different parameter estimates, control limits, and chart performance (i.e., different ARL values). We refer to this variation as *practitioner-to-practitioner* variability. Equivalently, this variation can be viewed as sampling variation for a single practitioner. Most often, charts are evaluated and the amount of Phase I data necessary for desired chart performance is determined based on the expected value of the ARL (AARL), averaging across the practitioner-to-practitioner variability.

The performance of the EWMA control chart with estimated parameters was first investigated by Jones et al. (2001), who derived the run length distribution of the chart. Jones et al. (2001) studied the run length distribution conditioned on specific values of the parameter estimates and also studied the unconditional run length distribution averaged over all possible values of the parameter estimates. They showed that the EWMA chart performance deteriorates substantially when parameters are estimated, particularly with small amounts of Phase I data. Similar to Quesenberry (1993), Jones et al. (2001) made sample-size recommendations based on the increase in the rate of early false alarms of a chart with estimated parameters over one with known parameters. This approach resulted in recommendations that more Phase I data are required for EWMA charts with small smoothing constants. Smaller values of the smoothing constant are typically recommended for detecting sustained shifts of smaller magnitude (Crowder (1987), Lucas and Saccucci (1990)).

Depending solely on the run length distribution averaged over all values of the parameter estimates does not reflect sampling variation or the amount of variation in the chart performance among practitioners. Although Jones et al. (2001) reported the standard deviation of the unconditional run length distribution (SDRL), that measure was also averaged over all possible values of the parameter estimates. Jones et al. (2001) additionally reported the unconditional 10th, 50th, and 90th percentiles of the run length distribution, which gives a better idea of how the EWMA chart performance varies according to the different values of the parameter estimates. It is difficult, however, to use multiple percentiles to make recommendations on the amount of Phase I data for control charts with estimated parameters. Our approach is to use the standard deviation of the ARL (SDARL) as a measure of the amount of practitioner-to-practitioner variability in control chart performance. Recently, several authors have used the SDARL as a metric for determining the necessary amount of Phase I data for control charts with estimated parameters (see, e.g., Jones and Steiner 2012; Zhang et al. 2013; Zhang et al. 2014; Lee et al. 2013; Aly et al. 2015; Saleh et al. 2015; and Faraz et al. 2015). These studies frequently show that impractically large amounts of Phase I data are needed for a practitioner to have confidence that his/her in-control ARL is near the desired value. The extent of this phenomenon was first recognized by Albers and Kallenberg (2004).

The findings of the studies accounting for the between-practitioner (or sampling) variability imply the necessity of having an alternative technique for controlling the chart performance. Recently, Jones and Steiner (2012) and Gandy and Kvaløy (2013) proposed a design procedure based on the bootstrap which guarantees, with a specified probability, a certain conditional performance for control charts. Their approach is to adjust the control limits such that $p\%$ of the in-control ARL values are at least a specified value; for example, at least 90% of the charts with a particular design would have in-control ARL values of 200 or more. The main objective of this approach is to limit the proportion of low in-control ARL values resulting from the use of insufficient amounts of Phase I data. Gandy and Kvaløy (2013) showed that even with the use of relatively small amounts of Phase I data, the out-of-control ARLs using this approach increase only slightly compared to the case when the standard design method is used.

In our article, we extend the work of Jones et al. (2001) by evaluating the performance of the EWMA chart with estimated parameters while considering the practitioner-to-practitioner variability using the standard deviation of the average run length (SDARL) metric. We also study the effect of the smoothing constant on the practitioner-to-practitioner variability. Because it has been shown that the standard deviation estimator has a strong effect on control chart performance (Saleh et al. (2015)), we further assess the performance of the EWMA chart using several estimators for the process standard deviation. Additionally, we design the EWMA chart using this bootstrap approach and investigate the effect of adjusting the control limits on the out-of-control performance of the chart.

In Section 2, we give an overview of the EWMA control chart with estimated parameters and present the estimators used for the in-control process parameters. In Section 3, we highlight the importance of incorporating the practitioner-to-practitioner variability when assessing the EWMA chart. In Section 4, we evaluate the EWMA chart in terms of the AARL, SDARL, and some percentiles of the ARL distribution. In Section 5, we investigate the in-control and out-of-control performance of the EWMA chart when the control limits are determined using the bootstrap approach. Finally, we give concluding remarks and recommendations in Section 6.

2. EWMA Chart with Estimated Parameters

We observe $X_{i1}, X_{i2}, \ldots, X_{in}$, $i = 1, 2, 3, \ldots$, independent random samples of size n at regular time intervals. For each sample, it is assumed that $X_{i1}, X_{i2}, \ldots, X_{in}$ are independent and identically distributed (i.i.d) normal random variables with mean μ and standard deviation σ. The objective is to detect any change in μ from its in-control value μ_0. We further assume that the in-control process standard deviation value is σ_0.

The EWMA chart statistic at time i is defined as

$$Z_i = \lambda \, \overline{X}_i + (1 - \lambda) Z_{i-1}, \tag{1}$$

where $\overline{X}_i$ is the ith sample mean and λ, $0 < \lambda \leq 1$, is a smoothing constant. The initial value Z_0 is usually set to be equal to the process target or to the estimate of the mean from the Phase I data. If $\lambda = 1$, the EWMA statistic is equal to the most recent sample mean, which is equivalent to the Shewhart $\overline{X}$-chart statistic. Under the normality assumption, Crowder (1987, 1989) and Lucas and Saccucci (1990) provided the optimal values of λ that correspond to different magnitudes of mean shifts. The EWMA chart signals when the statistic Z_i exceeds the limits given by

$$\mu_0 \pm L \sqrt{\frac{\lambda}{n(2-\lambda)}[1 - (1-\lambda)^{2i}]}\,\sigma_0, \tag{2}$$

where L is chosen to satisfy a specific in-control performance. The time-varying control limits in Equation (2) are the "exact" limits for the EWMA chart. As i increases, the term $(1-\lambda)^{2i}$ approaches zero and the limits in Equation (2) converge to the asymptotic limits given by

$$\mu_0 \pm L\sqrt{\frac{\lambda}{n(2-\lambda)}}\sigma_0. \tag{3}$$

For simplicity, we consider in our study the EWMA chart designed using the asymptotic limits defined in Equation (3).

Following a similar procedure to that of Jones et al. (2001), the chart statistic in Equation (1) can be rewritten as

$$Y_i = \lambda W_i + (1-\lambda)Y_{i-1}, \tag{4}$$

where W_i is the standardized sample mean defined as

$$W_i = \frac{\overline{X}_i - \mu_0}{\sigma_0/\sqrt{n}}, \quad i = 1, 2, 3, \ldots,$$

for any target mean value μ_0 and standard deviation σ_0. If μ_0 and σ_0 are unknown, they are typically replaced with their corresponding estimators to give

$$\hat{W}_i = \frac{\overline{X}_i - \hat{\mu}_0}{\hat{\sigma}_0/\sqrt{n}},$$

or equivalently

$$\hat{W}_i = \frac{1}{Q}\left(\nu_i + \gamma - \frac{Z}{\sqrt{m}}\right), \tag{5}$$

where $Q = \hat{\sigma}_0/\sigma_0$ is the ratio of the estimated in-control standard deviation to the actual in-control standard deviation, $\nu_i = \sqrt{n}(\overline{X}_i - (\mu_0 + \delta))/\sigma_0$ is the standardized Phase II sample mean with δ representing the mean shift, $\gamma = \sqrt{n}\delta/\sigma_0$ is the standardized mean shift, and $Z = \sqrt{mn}(\hat{\mu}_0 - \mu_0)/\sigma_0$ is the standardized difference between the actual in-control mean and the estimated in-control mean. If the process is in control, then $\gamma = \delta = 0$. We assume, without loss of generality, that $\mu_0 = 0$ and $\sigma_0/\sqrt{n} = 1$ and, because of standardization, the control limits in Equation (3) become

$$\pm L\sqrt{\frac{\lambda}{(2-\lambda)}}. \tag{6}$$

In our article, we consider estimating the in-control process mean μ_0 by the overall sample mean defined as

$$\hat{\mu}_0 = \frac{\sum_{i=1}^{m} \overline{X}_i}{m}, \tag{7}$$

where $\overline{X}_i$ is the ith Phase I sample mean. The process standard deviation, σ_0, is estimated by one of the following estimators:

$$\begin{aligned} \hat{\sigma}_1 &= \bar{R}/d_2(n), \\ \hat{\sigma}_2 &= \bar{S}/c_4(n), \\ \hat{\sigma}_3 &= S_{\text{pooled}}/c_4(v+1), \\ \hat{\sigma}_4 &= c_4(v+1)S_{\text{pooled}}, \\ \hat{\sigma}_5 &= S_{\text{pooled}}, \end{aligned} \tag{8}$$

where $\bar{R} = (\sum_{i=1}^{m} R_i)/m$, R_i is the ith Phase I sample range, $\bar{S} = (\sum_{i=1}^{m} S_i)/m$, S_i is the ith Phase I sample standard deviation, $S_{\text{pooled}} = \sqrt{(\sum_{i=1}^{m} S_i^2)/m}$, $v = m(n-1)$, and $c_4(\cdot)$ and $d_2(\cdot)$ are control chart constants. Tabulated values for c_4 and d_2 are widely available, e.g., in Montgomery (2013, p. 720). Each of the estimators $\hat{\sigma}_1$, $\hat{\sigma}_2$, and $\hat{\sigma}_3$ are unbiased estimators for σ_0, while $\hat{\sigma}_4$ and $\hat{\sigma}_5$ are biased.

Although the range-based estimator is easier for practitioners to calculate, it has the highest mean-squared error (MSE) among the estimators in Equation (8). Mahmoud et al. (2010) recommended that $\hat{\sigma}_1$ not be used in quality-control applications. Pooling the sample standard deviations provides lower values of MSE than averaging them (Derman and Ross (1995), Vardeman (1999), Mahmoud et al. (2010)). Among the different forms of the pooled estimator, Derman and Ross (1995) recommended the use of $\hat{\sigma}_5$, while Vardeman (1999) and Mahmoud et al. (2010) showed that $\hat{\sigma}_4$ has the lowest MSE.

3. Importance of Considering the Practitioner-to-Practitioner Variability

When the parameters are known, a control chart's ARL is a constant value; however, when the parameters are estimated, the ARL becomes a random variable due to the Phase I sampling. Control charts with estimated parameters have most often been evaluated in terms of the average ARL (AARL). The use of the AARL, however, does not reflect other important properties of the ARL. Because the ARL distribution can be skewed, the mean of the distribution (AARL) may not give an accurate measure of the location. More importantly, the AARL does not account for the variability in the ARL values. It is possible to have an AARL value close to the desired value, ARL_0, but with the individual ARL values widely dispersed. The larger the variability in the in-control ARL values among practitioners, the less confident one would be in a particular chart's performance. Basically, sampling variation affects each practitioner.

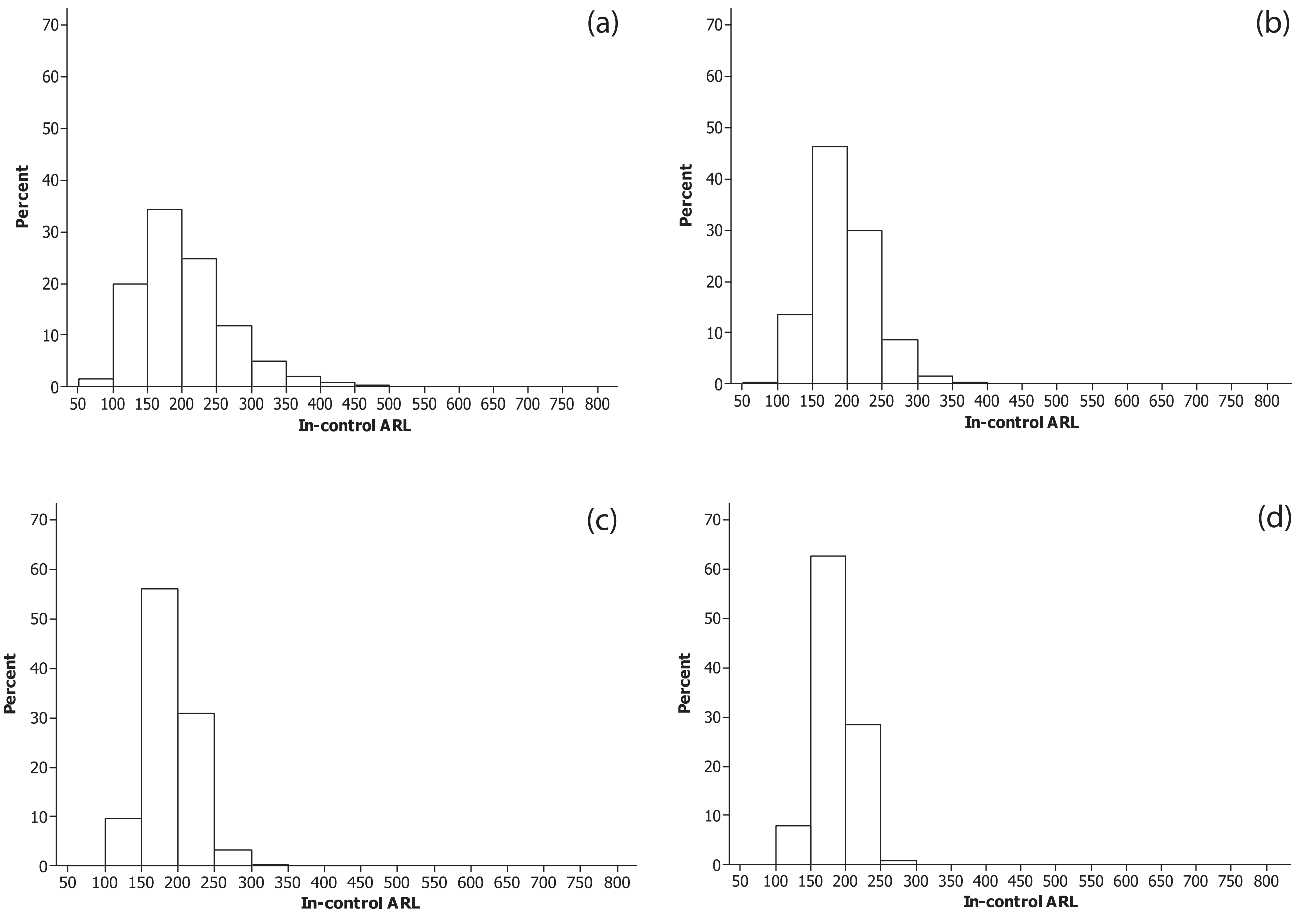

FIGURE 1. Relative Frequency Histograms of In-Control ARL Values Based on $\hat{\sigma}_3$ and $n = 5$. (a) $\lambda = 1.0$, $L = 2.807$, and $m = 100$; (b) $\lambda = 0.5$, $L = 2.777$, and $m = 200$; (c) $\lambda = 0.2$, $L = 2.636$, and $m = 300$; (d) $\lambda = 0.1$, $L = 2.454$, and $m = 400$.

Figure 1 presents relative frequency histograms of 100,000 simulated in-control ARL values for four EWMA smoothing constants based on Jones et al.'s (2001) sample-size recommendations. Table 1 provides the percentiles of these values. The standard deviation estimator used was $\hat{\sigma}_3$. The smoothing constant, λ, and the control chart constant, L, are those producing a chart with known parameters with a specified in-control ARL value of $\text{ARL}_0 = 200$. Jones et al. (2001) determined, when $n = 5$, that m should be of at least 400 if $\lambda = 0.1$, 300 if $\lambda = 0.2$, 200 if $\lambda = 0.5$, and 100 if $\lambda = 1.0$. Figure 1 shows that the in-control ARL values of charts designed using this amount of data are quite variable. Table 1 emphasizes this sampling variation. For example, if $\lambda = 0.1$ and 400 samples of size 5 are used to estimate the parameters, then 80% of the practitioners would have an in-control ARL value between

TABLE 1. Percentiles of the In-Control ARL Distribution Based on Jones et al.'s (2001) Sample Size Recommendations

λ	m	Min.	5th	10th	25th	50th	75th	90th	95th	Max.
1.0	100	52.6	116.6	129.7	155.8	191.5	236.4	287.3	324.0	774.8
0.5	200	71.0	133.9	144.4	164.2	189.7	219.8	251.4	272.1	487.1
0.2	300	64.6	140.0	150.6	167.8	187.5	208.6	229.2	242.3	369.9
0.1	400	66.9	142.6	153.6	170.3	186.9	203.1	218.2	227.9	311.6

153.6 and 218.2 and 90% between 142.6 and 227.9. A chart with an ARL of 143, for example, would give false signals more frequently than desired. Conversely, a chart with an ARL of 228 would give less frequent false signals than the value specified, but will be somewhat less sensitive to process changes. Notice that the median of the in-control ARL values corresponding to this case is 186.9, which is relatively close to 200.

The results in Figure 1 and Table 1 show the necessity of an alternative metric to measure the performance of control charts with estimated parameters. A straightforward measure of the practitioner-to-practitioner variability in control charts with estimated parameters is the standard deviation of the ARL (SDARL). The SDARL metric was proposed by Jones and Steiner (2012), who used it to determine the effect of the amount of Phase I data on the risk-adjusted cumulative sum (CUSUM) control chart. Saleh et al. (2015) evaluated the Shewhart $\bar{X}$- and the individuals X-control charts in terms of the SDARL metric. They concluded that accounting for the between-practitioner variability requires a far larger amount of Phase I data than that recommended by Quesenberry (1993) in order to reduce the variability among practitioners to an acceptable level. Also, Zhang et al. (2013, 2014) and Lee et al. (2013) used the SDARL metric in evaluating the performance of the exponential CUSUM chart, the geometric chart, and the Bernoulli CUSUM chart, respectively. Aly et al. (2015) used the SDARL metric in evaluating several different simple linear profile monitoring approaches when the in-control profile parameters are estimated. The use of the SDARL metric shows that the required amount of data to adequately reduce the variation in the in-control ARL to a reasonable level is often prohibitively large.

4. EWMA Performance Assessment

The in-control performance of the EWMA chart with estimated parameters was evaluated using the Markov chain approach described in Appendix A. The number of states used was 201. This number was found to balance a high level of accuracy and the acceptable time of computation. The calculations were performed using the SAS® statistical software, and results were validated using a Monte Carlo simulation. The process mean was estimated using the estimator in Equation (7) and the process standard deviation was estimated by each of the five estimators given in Equation (8). Different values of m, ranging from 30 to 5000, with sample sizes $n = 1$, 5, and 10 were considered. We used the same four combinations of control chart design parameters (λ, L) as considered by Jones et al. (2001): (0.1, 2.454), (0.2, 2.636), (0.5, 2.777), and (1.0, 2.807). Under the known in-control parameters assumption, these design parameters produce $\text{ARL}_0 = 200$.

Tables 2 and 3 display the in-control AARL and SDARL values, respectively, for each of the standard deviation estimators and values of m for samples of size $n = 5$. The last column in each table, $m = \infty$, refers to the case when the in-control process parameters are known. The bolded values correspond to the sample size recommendations of Jones et al. (2001) on the amount of Phase I data to use.

Although Jones et al.'s (2001) recommendations regarding the amount of Phase I data were based on reducing the occurrence of early false alarms, they also provided practitioners with AARL values close to ARL_0 as shown in Table 2. However, the results in Table 3 show that these values of m are associated with large values of the SDARL. Accounting for practitioner-to-practitioner variability in the ARL reveals that the recommended amount of data is not nearly large enough to ensure that individual practitioners will obtain an in-control ARL close to the specified value. Additionally, our results suggest that the larger the smoothing constant, the larger the SDARL will be for a given amount of data. For example, given $m = 30$, $\lambda = 0.1$, and the process standard deviation estimator $\hat{\sigma}_3$, the in-control AARL = 133.9 with SDARL = 81.2. If λ increases to 0.5 and 1.0, the in-control AARL increases to 183.6 and 212.3 and the corresponding SDARL increases as well to 123.5 and 142.7, respectively. Recall that, when $\lambda = 1$, the EWMA chart is equivalent to the Shewhart chart. Thus, we can conclude that Shewhart charts have higher levels of between-practitioner variability than the EWMA chart.

Another aspect of control chart performance that can be seen from Table 3 is the effect of the estimator of the process standard deviation on the control chart performance. For a given value of λ and small m, EWMA charts based on the standard deviation estimator $\hat{\sigma}_4$ have the smallest values of the SDARL.

In order to achieve stable in-control ARL performance when process parameters are estimated, the required amount of Phase I data should yield an in-control AARL value close to ARL_0 and an SDARL value that is sufficiently small. Zhang et al. (2014)

TABLE 2. In-Control AARL for Each Standard Deviation Estimator when m Phase I Samples, Each of Size $n = 5$, Are Used to Estimate the In-Control Values of the Process Parameters. Bolded values correspond to the sample size recommendations of Jones et al. (2001)

		m														
$\hat{\sigma}$	λ	30	50	100	200	300	400	500	600	700	800	900	1,000	3,000	5,000	∞
$\hat{\sigma}_1$	0.1	134.9	147.4	163.7	176.9	182.9	186.4	188.7	190.3	191.5	192.4	193.2	193.8	197.7	198.6	199.9
	0.2	153.4	162.8	175.2	184.9	189.2	191.6	193.1	194.2	195.0	195.6	196.1	196.5	198.9	199.5	200.3
	0.5	186.9	187.8	191.6	194.9	196.4	197.2	197.7	198.0	198.3	198.5	198.6	198.7	199.5	199.7	199.9
	1.0	216.7	208.2	203.5	201.6	201.0	200.8	200.6	200.5	200.4	200.3	200.3	200.3	200.1	200.0	200.0
$\hat{\sigma}_2$	0.1	134.4	147.1	163.5	176.8	182.9	186.4	188.6	190.3	191.5	192.4	193.2	193.8	197.7	198.6	199.9
	0.2	152.5	162.3	174.9	184.8	189.1	191.6	193.0	194.1	194.9	195.5	196.0	196.4	198.9	199.5	200.3
	0.5	185.2	187.0	191.2	194.7	196.2	197.2	197.6	197.9	198.2	198.4	198.6	198.7	199.5	199.6	199.9
	1.0	214.4	207.1	202.9	201.3	200.8	200.7	200.5	200.4	200.3	200.3	200.2	200.2	200.1	200.0	200.0
$\hat{\sigma}_3$	0.1	133.9	146.8	163.3	176.7	182.8	**186.3**	188.6	190.2	191.4	192.4	193.1	193.7	197.7	198.6	199.9
	0.2	151.6	161.7	174.6	184.6	**189.0**	191.4	193.0	194.1	194.9	195.5	196.0	196.4	198.9	199.5	200.3
	0.5	183.6	186.1	190.7	**194.5**	196.1	196.9	197.5	197.8	198.1	198.3	198.5	198.6	199.5	199.6	199.9
	1.0	212.3	206.0	**202.4**	201.0	200.7	200.5	200.4	200.3	200.2	200.2	200.2	200.1	200.0	200.0	200.0
$\hat{\sigma}_4$	0.1	131.0	144.9	162.2	176.1	182.4	186.0	188.3	190.0	191.2	192.2	193.0	193.6	197.7	198.5	199.9
	0.2	147.5	159.1	173.1	183.8	188.4	191.0	192.6	193.8	194.6	195.3	195.8	196.2	198.9	199.4	200.3
	0.5	177.3	182.3	188.8	193.5	195.4	196.4	197.1	197.5	197.8	198.1	198.3	198.4	199.4	199.6	199.9
	1.0	204.4	201.4	200.2	199.9	199.9	199.9	199.9	199.9	199.9	199.9	199.9	199.9	199.9	199.9	200.0
$\hat{\sigma}_5$	0.1	132.4	145.9	162.7	176.4	182.6	186.1	188.5	190.1	191.3	192.3	193.1	193.7	197.7	198.6	199.0
	0.2	149.6	160.4	173.9	184.2	188.7	191.2	192.8	193.9	194.7	185.4	195.9	196.3	198.9	199.4	200.3
	0.5	180.4	184.2	189.7	194.0	195.7	196.7	197.3	197.7	198.0	198.2	198.4	198.5	199.4	199.6	199.9
	1.0	208.3	203.7	201.3	200.5	200.3	200.2	200.1	200.1	200.1	200.1	200.1	200.0	200.0	200.0	200.0

suggested that an SDARL within 10% of the ARL_0 may be reasonable, although still reflecting a significant amount of variation. Consequently, based on our results, a practitioner would need about 600 samples of size $n = 5$ if $\lambda = 0.1$, 700 if $\lambda = 0.2$, 900 if $\lambda = 0.5$, and 1000 if $\lambda = 1.0$ to obtain SDARL values of no more than 20 (10% of 200). These recommendations hold if any of the standard deviation estimators is used except for the estimator $\hat{\sigma}_1$. If $\hat{\sigma}_1$ is used, a practitioner would need a larger amount of Phase I data; 700 samples of size $n = 5$ if $\lambda = 0.1$, 800 if $\lambda = 0.2$, and 1000 if $\lambda = 0.5$ or 1.0. In most applications, it will not be realistic to obtain this amount of stable Phase I data from the process.

Tables 4, 5, and 6 provide practitioners with the in-control ARL percentiles for different values of m, of a fixed sample size $n = 5$ when $(\lambda, L) = (0.1, 2.454)$, (0.2, 2.636), and (0.5, 2.777), respectively. The values presented were calculated using 20,000 simulated in-control ARL values when the process standard deviation is estimated by $\hat{\sigma}_3$. We also show the minimum and maximum values obtained in our simulations. These tables may help practitioners to assess the performance of their chart according to their available amount of Phase I data.

Furthermore, we studied the required amount of Phase I data of various values of the intended in-control ARL (ARL_0). Tables 7–8 display the in-control AARL and SDARL values for different values of ARL_0 when $\lambda = 0.1$ and 0.5, respectively. The last row in each table, entitled $m = \infty$, refers to the case when the in-control process parameters are known. The bolded and italicized SDARL values in Tables 7–8 are those that have an SDARL value within 10% of ARL_0. As shown, the required number of samples m increases with an increase in the ARL_0 value.

TABLE 3.In-Control SDARL for Each Standard Deviation Estimator when m Phase I Samples, Each of Size $n = 5$, Are Used to Estimate the In-Control Values of the Process Parameters. Bolded values correspond to the sample size recommendations of Jones et al. (2001)

		m														
$\hat{\sigma}$	λ	30	50	100	200	300	400	500	600	700	800	900	1,000	3,000	5,000	∞
$\hat{\sigma}_1$	0.1	84.5	69.5	52.8	38.2	31.0	26.5	23.4	21.1	19.3	17.9	16.8	15.7	8.4	6.4	0.0
	0.2	102.2	79.0	56.9	40.2	32.6	28.0	24.8	22.5	20.7	19.3	18.2	17.1	9.6	7.4	0.0
	0.5	133.8	95.4	64.7	44.7	36.2	31.2	27.8	25.3	23.4	21.9	20.7	19.5	11.2	8.7	0.0
	1.0	156.2	106.5	69.7	47.4	38.3	33.0	29.4	26.8	24.7	23.1	21.8	20.6	11.8	9.2	0.0
$\hat{\sigma}_2$	0.1	82.8	68.6	52.1	37.8	30.6	25.9	23.1	20.8	19.0	17.6	16.4	15.5	8.3	6.2	0.0
	0.2	99.2	77.5	55.9	39.5	32.0	27.1	24.3	22.1	20.3	18.9	17.7	16.8	9.4	7.2	0.0
	0.5	128.4	92.9	63.0	43.6	35.4	30.2	27.2	24.8	22.9	21.4	20.1	19.1	11.0	8.4	0.0
	1.0	149.0	103.3	67.7	46.2	37.4	31.9	28.7	26.1	24.1	22.6	21.2	20.2	11.6	8.9	0.0
$\hat{\sigma}_3$	0.1	81.2	67.6	51.4	37.3	30.2	**25.9**	22.7	20.4	18.7	17.4	16.0	15.2	8.1	6.1	0.0
	0.2	96.6	75.8	54.8	38.7	**31.2**	26.9	23.8	21.6	19.9	18.6	17.3	16.4	9.2	7.0	0.0
	0.5	123.5	90.1	61.4	**42.5**	34.3	29.7	26.5	24.1	22.3	20.9	19.5	18.6	10.7	8.2	0.0
	1.0	142.7	99.7	**65.7**	45.0	36.2	31.3	27.9	25.4	23.5	22.0	20.6	19.6	11.3	8.7	0.0
$\hat{\sigma}_4$	0.1	78.8	66.3	50.9	37.1	30.0	25.7	22.7	20.4	18.7	17.4	16.0	15.2	8.1	6.1	0.0
	0.2	93.0	74.2	54.2	38.5	31.1	26.8	23.8	21.5	19.9	18.5	17.2	16.4	9.2	7.0	0.0
	0.5	118.1	87.7	60.6	42.3	34.2	29.6	26.4	24.1	22.3	20.9	19.5	18.6	10.7	8.2	0.0
	1.0	136.0	97.0	64.9	44.7	36.1	31.2	27.8	25.4	23.5	22.0	20.6	19.6	11.3	8.7	0.0
$\hat{\sigma}_5$	0.1	80.0	67.0	51.2	37.2	30.0	25.7	22.7	20.4	18.7	17.4	16.0	15.2	8.1	6.1	0.0
	0.2	94.8	75.0	54.5	38.6	31.1	26.8	23.8	21.6	19.9	18.5	17.2	16.4	9.2	7.0	0.0
	0.5	120.8	88.9	61.0	42.4	34.3	29.7	26.5	24.1	22.3	20.9	19.5	18.6	10.7	8.2	0.0
	1.0	139.3	98.3	65.3	44.8	36.1	31.3	27.9	25.4	23.5	22.0	20.6	19.6	11.3	8.7	0.0

For example, an EWMA chart with $\lambda = 0.1$ requires about 400 in-control samples of size $n = 5$ when $ARL_0 = 100$, but this increases to 1000 samples of size $n = 5$ when $ARL_0 = 500$. This phenomenon occurs because the larger the in-control ARL, the wider the control limits and the further the estimated con-

TABLE 4. Percentiles of the In-Control ARL Distribution for $\lambda = 0.1$ when m Phase I Samples, Each of Size $n = 5$, Are Used to Estimate the In-Control Values of the Process Parameters and the Standard Deviation Estimator Is $\hat{\sigma}_3$

m	Min.	5th	10th	25th	50th	75th	90th	95th	Max.
50	14.2	52.4	65.6	96.7	139.6	186.7	235.8	268.4	620.1
100	26.2	81.1	96.9	127.7	161.9	195.9	228.4	250.3	471.2
200	45.7	112.3	127.4	152.5	176.8	201.0	223.0	236.5	342.0
400	66.9	142.6	153.6	170.3	186.9	203.1	218.2	227.9	311.6
600	94.2	155.8	164.6	177.4	190.6	203.6	215.7	223.1	304.9
800	114.4	163.7	170.9	181.5	192.4	203.5	214.1	220.2	264.4
1,000	118.1	168.4	174.5	183.8	193.7	203.7	212.6	218.2	257.5
2,000	150.1	180.2	184.0	190.1	196.1	203.3	209.5	213.3	238.9

TABLE 5. Percentiles of the In-Control ARL Distribution for $\lambda = 0.2$ when m Phase I Samples, Each of Size $n = 5$, Are Used to Estimate the In-Control Values of the Process Parameters and the Standard Deviation Estimator Is $\hat{\sigma}_3$

m	Min.	5th	10th	25th	50th	75th	90th	95th	Max.
50	19.5	63.1	78.3	109.4	150.0	201.5	258.9	301.5	1,110.7
100	36.0	93.7	109.1	136.4	168.9	206.0	245.5	271.6	527.7
300	64.6	140.0	150.6	167.8	187.5	208.6	229.2	242.3	369.9
500	95.5	155.6	163.4	176.7	191.9	208.1	223.2	233.0	304.4
700	105.7	163.6	170.3	181.4	194.2	207.6	220.6	228.7	296.7
1,000	136.1	170.2	175.8	185.2	195.7	207.0	217.5	224.1	267.5
2,000	149.3	180.2	184.1	190.7	198.0	205.8	213.0	217.5	250.5
5,000	174.0	188.1	190.5	194.7	199.4	204.2	208.6	211.4	227.9

TABLE 6. Percentiles of the In-Control ARL Distribution for $\lambda = 0.5$ when m Phase I Samples, Each of Size $n = 5$, Are Used to Estimate the In-Control Values of the Process Parameters and the Standard Deviation Estimator Is $\hat{\sigma}_3$

m	Min.	5th	10th	25th	50th	75th	90th	95th	Max.
50	27.3	81.4	95.5	125.1	168.6	227.8	301.3	358.3	1,012.2
200	71.0	133.9	144.4	164.2	189.7	219.8	251.4	272.1	487.1
500	115.2	157.6	164.9	178.8	195.6	214.1	232.0	243.7	334.4
700	125.1	163.9	170.7	182.5	196.9	212.2	227.0	236.6	314.0
900	128.6	168.2	174.1	184.8	197.3	210.9	224.0	232.3	335.6
1,000	138.4	169.7	175.5	185.7	197.5	210.4	223.1	231.0	285.3
3,000	161.4	182.4	186.1	192.3	199.2	206.7	213.5	217.5	244.8
5,000	170.2	186.4	189.0	193.8	199.4	205.1	210.4	213.5	235.0

TABLE 7. In-Control AARL and SDARL Values for Different ARL_0 for $\lambda = 0.1$ when m Phase I Samples, Each of Size $n = 5$, Are Used to Estimate the In-Control Values of the Process Parameters and the Standard Deviation Estimator Is $\hat{\sigma}_3$

	$ARL_0 = 100$ ($L = 2.148$)		$ARL_0 = 200$ ($L = 2.454$)		$ARL_0 = 370$ ($L = 2.702$)		$ARL_0 = 500$ ($L = 2.815$)	
m	AARL	SDARL	AARL	SDARL	AARL	SDARL	AARL	SDARL
50	78.6	28.0	146.8	67.6	258.2	144.9	341.1	208.9
100	86.0	20.8	163.3	51.4	290.2	111.2	384.6	160.4
300	93.9	11.7	182.8	30.1	331.1	67.0	442.2	97.8
400	95.2	***10.0***	186.3	25.9	338.8	57.7	453.4	84.4
500	96.1	8.8	188.6	22.7	344.0	51.1	460.9	74.9
600	96.7	7.9	190.2	***20.4***	347.7	46.1	466.3	67.7
700	97.1	7.2	191.4	18.7	350.5	42.3	470.4	62.1
800	97.4	6.7	192.4	17.4	352.7	39.2	473.6	57.7
900	97.7	6.2	193.1	16.0	354.5	***36.4***	476.2	53.6
1,000	97.9	5.9	193.7	15.2	355.9	34.4	478.3	***50.7***
1,100	98.1	5.5	194.3	14.4	357.1	32.5	480.1	47.9
∞	100.1	0.0	199.9	0.0	370.7	0.0	500.5	0.0

TABLE 8. In-Control AARL and SDARL Values for Different ARL_0 for $\lambda = 0.5$ when m Phase I Samples, Each of Size $n = 5$, Are Used to Estimate the In-Control Values of the Process Parameters and the Standard Deviation Estimator Is $\hat{\sigma}_3$

	$ARL_0 = 100$ ($L = 2.534$)		$ARL_0 = 200$ ($L = 2.777$)		$ARL_0 = 370$ ($L = 2.978$)		$ARL_0 = 500$ ($L = 3.071$)	
m	AARL	SDARL	AARL	SDARL	AARL	SDARL	AARL	SDARL
50	93.1	37.0	186.1	90.1	347.1	196.5	470.4	285.9
100	95.7	25.5	190.7	61.4	353.6	131.6	477.4	189.7
500	98.9	11.1	197.5	26.5	365.7	56.3	493.3	80.8
600	99.1	***10.1***	197.8	24.1	366.5	51.3	494.3	73.5
700	99.2	9.4	198.1	22.3	367.0	47.4	495.0	68.0
900	99.4	8.2	198.5	***19.5***	367.8	41.6	496.0	59.6
1,000	99.4	7.8	198.6	18.6	368.0	39.5	496.4	56.7
1,100	99.5	7.4	198.7	17.7	368.2	***37.6***	496.7	53.9
1,200	99.5	7.1	198.8	17.0	368.4	36.0	496.9	51.6
1,300	99.6	6.8	198.9	16.3	368.6	34.6	497.1	***49.6***
∞	100.0	0.0	199.9	0.0	370.5	0.0	499.9	0.0

trol limits are in the tails of the distribution of the control chart statistic. It is well-known that estimating more extreme quantiles of a distribution requires larger samples to achieve the same precision as when estimating more central quantiles.

To study the effect of the sample size n, we considered the in-control AARL and SDARL values for $n = 1$ and $n = 10$. In each case, the control limits for 10,000 charts were estimated. Markov chains were used to approximate the ARL for $n = 10$ and simulation with 10,000 run lengths were used to estimate each in-control ARL when $n = 1$. The results are given in Tables 9 and 10 for $n = 10$. We restrict our attention to the two most efficient estimators of the standard deviation. These tables show that the data requirements, in terms of the total number of observations, are similar as for the case $n = 5$.

Tables 11 and 12 contain the in-control values of AARL and SDARL for the EWMA chart when $n = 1$ and the process standard deviation is estimated by the moving-range estimator, defined as

$$\text{MR} = \frac{\overline{\text{MR}}}{1.128} = \frac{1}{1.128}\frac{1}{m-1}\sum_{i=2}^{m}|X_i - X_{i-1}|.$$

Again, as with $n = 5$ and $n = 10$, several thousand observations are needed for the in-control SDARL

TABLE 9. In-Control AARL Values for the EWMA Chart with $n = 10$ and $ARL_0 = 200$

		m							
$\hat{\sigma}$	λ	30	50	100	200	500	1,000	5,000	∞
$\hat{\sigma}_3$	0.1	129.8	143.8	161.7	175.7	188.2	193.4	198.5	200.0
	0.2	143.6	156.4	171.7	183.0	192.3	196.1	199.3	200.0
	0.5	168.4	176.5	186.0	192.0	196.6	198.1	199.5	200.0
	1.0	192.5	193.8	196.6	198.3	199.3	199.8	199.9	200.0
$\hat{\sigma}_4$	0.1	127.1	143.0	160.9	175.5	188.1	193.5	198.5	200.0
	0.2	141.8	154.9	170.6	182.7	191.9	195.8	199.4	200.0
	0.5	166.3	175.7	185.2	191.4	196.3	198.1	199.5	200.0
	1.0	189.5	191.8	195.5	197.6	199.0	199.5	199.8	200.0

TABLE 10. In-Control SDARL Values for the EWMA Chart with $n = 10$ and $ARL_0 = 200$

		m							
$\hat{\sigma}$	λ	30	50	100	200	500	1,000	5,000	∞
$\hat{\sigma}_3$	0.1	66.2	56.9	44.1	31.5	18.8	12.0	4.3	0.0
	0.2	70.7	57.8	43.0	30.2	17.7	11.8	4.8	0.0
	0.5	76.8	59.4	42.5	29.1	18.1	12.4	5.5	0.0
	1.0	81.7	60.7	42.3	29.5	18.5	13.2	5.8	0.0
$\hat{\sigma}_4$	0.1	64.5	56.5	43.8	31.6	18.4	11.8	4.3	0.0
	0.2	69.7	57.4	42.7	30.0	17.8	11.7	4.8	0.0
	0.5	75.3	59.3	42.2	29.1	18.0	12.6	5.5	0.0
	1.0	79.7	60.0	42.6	29.5	18.5	13.0	5.8	0.0

values to be relatively small, say within 10% of the desired in-control ARL value of 200. One has very little to no control over the in-control ARL value if one follows the common recommendation of 25–50 individual observations in Phase I. This result was also demonstrated by Saleh et al. (2015) for $\lambda = 1$.

We also investigated the required number of Phase I individual observations when changing the value of the desired in-control ARL for the EWMA chart. Table 13 and Table 14 contain our results when $\lambda = 0.1$ and $\lambda = 0.5$, respectively. The bolded and italicized values can be used to identify the number of observations required to have the SDARL value be within 10% of the desired in-control ARL_0 value. Higher numbers of observations are required for the larger value of λ. In addition, the required number of observations increases as the desired value of the in-control ARL_0 increases.

5. Adjusting the EWMA Control Limits

In order to overcome the problem of the often low in-control ARL values when using estimated parameters, Jones and Steiner (2012) and Gandy and

TABLE 11. In-Control AARL Values for the EWMA Chart with $n = 1$ and $ARL_0 = 200$

		m							
$\hat{\sigma}$	λ	30	50	100	200	500	1,000	5,000	∞
MR	0.1	248.5	192.6	183.6	184.0	194.1	196.8	199.0	200.0
	0.2	352.6	247.7	212.0	201.6	199.8	199.4	200.3	200.0
	0.5	721.0	365.1	258.0	223.9	208.8	204.9	200.5	200.0
	1.0	976.4	458.8	275.0	233.5	210.1	206.3	200.3	200.0

TABLE 12. In-Control SDARL Values for the EWMA Chart with $n = 1$ and $ARL_0 = 200$

		m							
$\hat{\sigma}$	λ	30	50	100	200	500	1,000	5,000	∞
MR	0.1	2,061.0	234.7	123.3	80.6	51.3	33.0	15.1	0.0
	0.2	1,517.5	439.9	181.8	100.4	60.0	40.8	17.9	0.0
	0.5	3,833.3	1,228.1	307.8	144.8	75.1	48.4	21.9	0.0
	1.0	5,622.3	1,301.7	299.9	159.3	79.1	54.7	22.9	0.0

TABLE 13. In-Control AARL and SDARL Values for Different ARL_0 for $\lambda = 0.1$ when *m* Phase I Samples, Each of Size $n = 1$, Are Used to Estimate the In-Control Values of the Process Parameters and the Standard Deviation Estimator Is MR

	$ARL_0 = 100$ ($L = 2.148$)		$ARL_0 = 200$ ($L = 2.454$)		$ARL_0 = 370$ ($L = 2.702$)		$ARL_0 = 500$ ($L = 2.815$)	
m	AARL	SDARL	AARL	SDARL	AARL	SDARL	AARL	SDARL
100	92.3	46.3	183.6	123.3	372.3	395.6	482.7	459.3
250	95.2	27.4	189.1	69.9	349.5	165.1	474.4	246.8
500	97.0	20.0	194.1	51.3	362.0	109.5	482.8	163.8
1,000	98.5	13.7	196.8	33.0	363.5	76.5	494.1	122.3
2,000	98.8	***9.6***	197.0	23.9	366.8	56.5	488.1	78.5
3,000	99.1	8.0	197.3	***19.6***	369.6	43.9	496.7	65.6
4,000	99.0	6.7	199.2	17.1	369.1	37.9	497.6	54.5
5,000	98.8	6.0	199.0	15.1	369.1	***35.1***	498.0	50.3
6,000	98.9	5.6	198.0	13.7	371.9	32.4	499.7	***47.3***
∞	100.0	0.0	200.0	0.0	370.0	0.0	500.0	0.0

Kvaløy (2013) argued that determining the control limits should be based on the conditional in-control ARL instead of the unconditional one. Their proposal was to adjust the control limits in a way that guarantees, with a suitably high prespecified probability, that the conditional in-control ARL meets or exceeds the desired level.

Gandy and Kvaløy's (2013) approach is based on bootstrapping the Phase I data to construct an approximate confidence interval for the control limits. The general bootstrap procedure, introduced by Efron (1979), is a resampling technique used to estimate the sampling distribution of any sample statistic. In quality-control applications, control charts designed based on bootstrap methods have been suggested as alternatives for the standard design methods. See, for example, Bajgier (1992), Seppala et al. (1995), Liu and Tang (1996), and Jones and Woodall (1998). Recently, Chatterjee and Qiu (2009) prop-

TABLE 14. In-Control AARL and SDARL Values for Different ARL_0 for $\lambda = 0.5$ when *m* Phase I Samples, Each of Size $n = 1$, Are Used to Estimate the In-Control Values of the Process Parameters and the Standard Deviation Estimator Is MR

	$ARL_0 = 100$ ($L = 2.534$)		$ARL_0 = 200$ ($L = 2.777$)		$ARL_0 = 370$ ($L = 2.978$)		$ARL_0 = 500$ ($L = 3.071$)	
m	AARL	SDARL	AARL	SDARL	AARL	SDARL	AARL	SDARL
100	118.6	117.2	258.0	307.8	502.3	700.2	747.4	1,174.0
250	104.6	45.9	226.2	132.5	416.4	286.9	563.5	365.4
500	101.9	30.1	208.8	75.1	395.9	165.3	533.6	231.6
1,000	101.4	20.9	204.9	48.4	379.9	103.0	517.4	152.8
2,000	99.7	14.2	200.9	34.3	375.0	72.8	509.9	106.6
3,000	99.4	11.4	201.2	27.6	371.4	60.3	506.0	88.0
4,000	99.4	***10.0***	199.1	23.6	373.3	50.8	504.1	71.7
5,000	99.2	8.8	200.5	21.9	372.8	45.1	502.0	64.4
6,000	99.3	8.1	200.0	***19.6***	373.2	43.3	500.7	60.7
7000	99.5	7.7	200.2	18.4	370.9	***37.1***	500.2	***49.1***
∞	100.0	0.0	200.0	0.0	370.0	0.0	500.0	0.0

osed estimating the control limits of the CUSUM chart using the bootstrap. Prior work on the bootstrap methods used in quality control focused on determining estimated control limits, not on controlling the conditional ARL performance of control charts.

In order to best describe Gandy and Kvaløy's (2013) approach, let us first define P as the true in-control distribution, $\hat{P}$ as the estimated in-control distribution, $\theta = (\mu, \sigma)$ as the vector of process parameters, $\hat{\theta} = (\hat{\mu}, \hat{\sigma})$ as the vector of estimated process parameters, and q as the control chart limit satisfying a specific in-control ARL. The quantities $\hat{P}$ and $\hat{\theta}$ are obtained from m in-control Phase I samples each of size n. The quantity q is a function of P and θ or their estimates. For example, $q(P, \hat{\theta})$ represents the value of control limits conditioned on $\hat{\theta}$ under the true in-control distribution P. For simplicity, in this study, we evaluate a limit q for the absolute value of the EWMA chart statistic, defined in Equation (4), divided by the quantity $\sqrt{\lambda/(2-\lambda)}$. Therefore, the control limit q that produces the desired in-control ARL is equal to the value of L defined in Equation (6).

When parameters are unknown, the observed control chart performance depends on $q(P, \hat{\theta})$, which is unknown because P is unobservable. Gandy and Kvaløy (2013) proposed using the estimator $q(\hat{P}, \hat{\theta})$ to build a lower one-sided confidence interval for $q(P, \hat{\theta})$ using the bootstrap technique. Let $(1-\alpha^*)\%$ be the percent of the in-control ARL values equal to or higher than the ARL_0, then we can write

$$P(q(\hat{P}, \hat{\theta}) - q(P, \hat{\theta}) > p_{\alpha^*}) = P(q(P, \hat{\theta}) < q(\hat{P}, \hat{\theta}) - p_{\alpha^*}) = 1 - \alpha^*, \quad (9)$$

where p_{α^*} is a constant. The quantity p_{α^*} is unknown because it represents the (α^*) quantile of the unobserved sampling distribution of $q(\hat{P}, \hat{\theta}) - q(P, \hat{\theta})$. Note that Gandy and Kvaløy (2013) incorrectly referred to p_{α^*} as the $(1-\alpha^*)$ quantile. This was a typographical error because it should be the α^* quantile. Gandy and Kvaløy (2013) proposed using the bootstrap technique to estimate the distribution of $q(\hat{P}, \hat{\theta}) - q(P, \hat{\theta})$ with the distribution of $q(\hat{P}^*, \hat{\theta}^*) - q(\hat{P}, \hat{\theta}^*)$, where $\hat{P}^*$ and $\hat{\theta}^* = (\hat{\mu}^*, \hat{\sigma}^*)$ are the estimated in-control distribution and process parameters from the bootstrap samples, respectively. If B is the number of bootstrap samples, then p_{α^*} is approximated with $p^*_{\alpha^*}$, which represents the (α^*) quantile of $[q(\hat{P}^*_i, \hat{\theta}^*_i) - q(\hat{P}, \hat{\theta}^*_i)]$, $i = 1, 2, 3, \ldots, B$. The upper bound $q(\hat{P}, \hat{\theta}) - p^*_{\alpha^*}$ is then taken as the adjusted control limit.

The simulation steps followed in our article are the same as those listed in Gandy and Kvaløy (2013, p. 651). In our simulation procedure, we used $m = 50$ samples of size $n = 5$, $\alpha^* = 0.1$, $\lambda = 0.1$, $B = 1{,}000$ bootstrap samples, and the process standard deviation estimator $\hat{\sigma}_3$. We assumed, without loss of generality, that the unknown true in-control distribution is $N(0, \sqrt{n})$. We assumed that the desired in-control ARL_0 is 200. Because we found that the Shewhart chart has higher levels of between-practitioner variability than the EWMA chart, we additionally designed it using this bootstrap approach. The same simulation settings were used for the Shewhart chart. The procedure followed in calculating the control limits, $q(\hat{P}, \hat{\theta})$, $q(\hat{P}^*_i, \hat{\theta}^*_i)$, and $q(\hat{P}, \hat{\theta}^*_i)$, for each of the Shewhart and EWMA control charts is discussed in detail in Appendix B. Once the limit $(q(\hat{P}, \hat{\theta}) - p^*_{\alpha^*})$ was determined, the corresponding in-control and out-of-control ARLs were calculated. For the EWMA chart, the Markov chain approach described in Appendix A was used in calculating the ARL.

Figures 2–3 contain the boxplots of the in-control and out-of-control ARL distributions, respectively, for the EWMA and Shewhart control charts. For both the EWMA and Shewhart charts, the limits computed with the bootstrap adjustment are indicated as "Adjusted Limits". For reference, charts with "Unadjusted Limits" were computed with $m = 50$ samples of size $n = 5$ using $(\lambda, L) = (0.1, 2.454)$ for EWMA charts and $L = 2.807$ for Shewhart charts. The out-of-control ARL values were computed with a mean shift of $\delta = 1$, and the boxplots were constructed from 2000 ARL values. In Figure 2, one can see, as expected, that the adjusted limits resulted in about 90% of the in-control ARL values for both the EWMA and Shewhart charts of at least 200 when computed using the bootstrap approach. Interestingly, more than 75% of the EWMA charts and 50% of the Shewhart charts with unadjusted limits had an in-control ARL below 200, indicating a higher incidence of false alarms.

An interesting feature of Figure 2 is that the EWMA charts based on the bootstrap design have a much more variable in-control ARL distribution than the charts based on unadjusted limits. Although the in-control ARL distribution of the EWMA chart is extremely skewed to the right and more variable than that of the unadjusted limits, the out-of-control ARL distribution of the EWMA chart with the adjusted limits is very tight, as shown in Figure 3. The EWMA design based on the bootstrap approach has

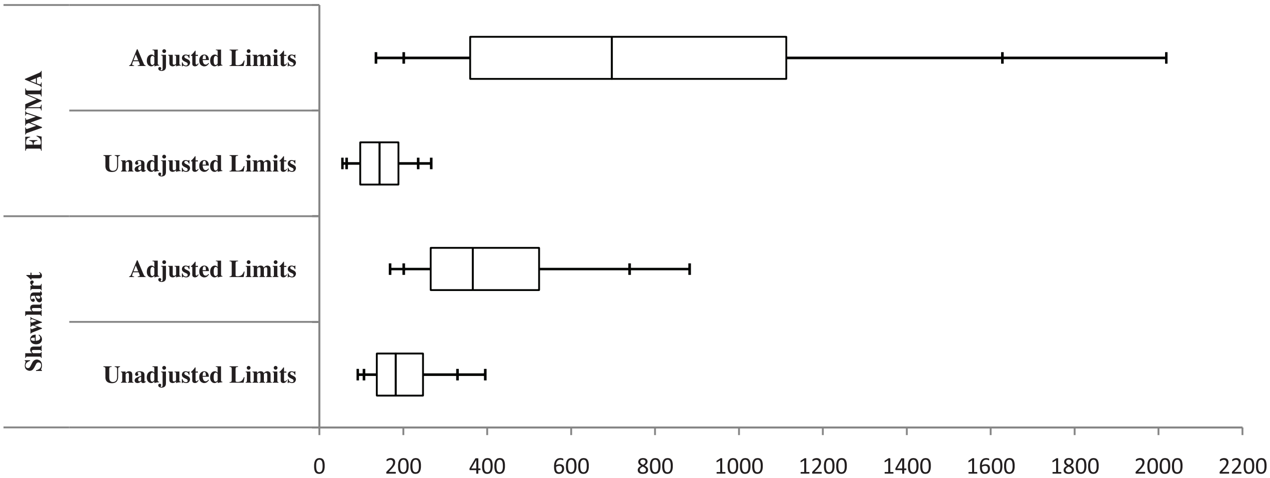

FIGURE 2. In-Control Distribution of the Conditional ARL when $m = 50$ and $n = 5$. The boxplots show the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentiles of the conditional in-control ARL distribution.

a slightly more variable out-of-control ARL distribution than the standard design. The median out-of-control ARL is around 12 for the adjusted limits and 9 for the unadjusted limits. This small loss in out-of-control performance comes with "guaranteed" in-control performance with 90% of the bootstrap adjusted charts having in-control ARL values above 200 as compared with only 25% of the charts with unadjusted limits. Although the increased variability in the in-control ARL distribution of the EWMA charts based on the adjusted limits was initially surprising to us, we quickly realized that we are not too concerned about charts with large in-control ARL values as long as they can quickly detect an out-of-control event.

Another interesting feature of Figures 2 and 3 is that the out-of-control ARL values of the Shewhart chart with the adjusted limits are considerably higher than those of the EWMA chart with either adjusted or unadjusted limits. Hence, if the goal is to avoid frequent false alarms and to detect this sustained shift quickly, then the EWMA chart remains much preferred to the Shewhart chart.

Figure 4 shows the relationship between the in-control ARL values and the out-of-control ARL val-

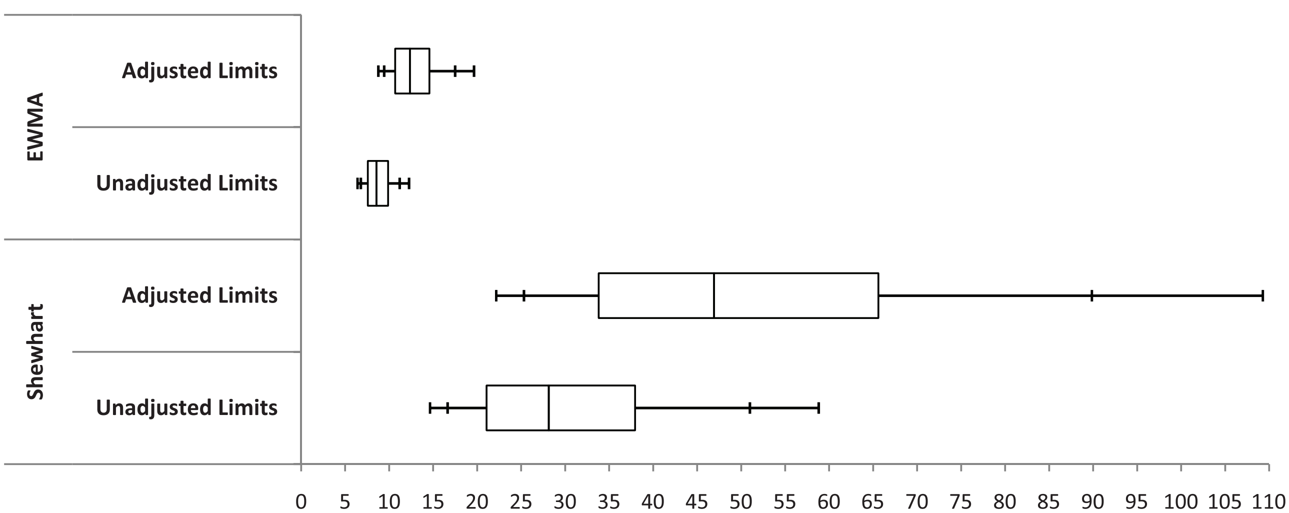

FIGURE 3. Out-of-Control Distribution of the Conditional ARL when $m = 50$, $n = 5$, and a mean shift $\delta = 1$. The boxplots show the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentiles of the conditional out-of-control ARL distribution.

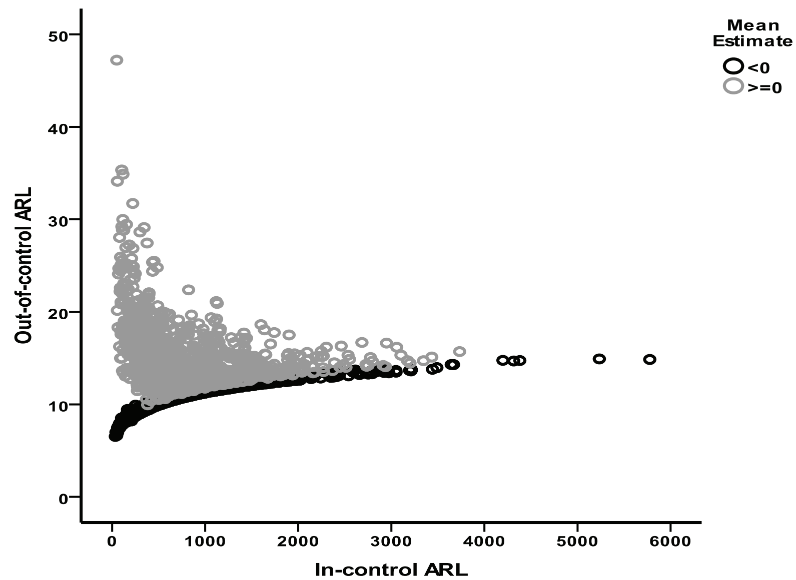

FIGURE 4. Scatterplot of the In-Control ARL Values vs. the Out-of-Control ARL Values of the EWMA Control Chart Categorized by the Mean Estimates Overestimating or Underestimating the Process Mean.

ues of the EWMA chart. The scatterplot presents the out-of-control ARL versus the in-control ARL, categorized by the standardized mean being underestimated (< 0) or overestimated (≥ 0). The lower smooth part of the scatterplot represents the case when the process mean is underestimated. Unexpectedly, the high out-of-control ARL values are associated with the lowest in-control ARL values. It can be concluded from this figure that the increase in the out-of-control ARL is due to overestimating the process mean rather than having a higher in-control ARL. Another point to note from Figure 4 is that a positive sustained shift along with an underestimated in-control mean increases the effective shift size and, as a consequence, results in a low out-of-control ARL value. Overestimating the in-control mean, on the other hand, leads to a decrease in the effective shift size and, thus, a significant increase in the out-of-control ARL.

6. Concluding Remarks

In our article, we have extended the work of Jones et al. (2001) by using the SDARL metric in evaluating the in-control performance of the EWMA control chart when the parameters are estimated. Accounting for the practitioner-to-practitioner variability led to some quite different conclusions regarding the chart performance. First, the EWMA chart requires more Phase I data than previously recommended in order to have consistent chart performance among practitioners. Additionally, we found that charts designed with large values of the EWMA smoothing constant have more variability in the ARL distribution; thus, we recommend more Phase I data be used with larger smoothing constants. Because EWMA charts are typically used when quickly detecting small sustained shifts is of interest, the charts are most often designed with small values of the smoothing constant ($\lambda < 0.25$).

With our recommendations regarding the required amount of Phase I data, we can easily see the difficulty in controlling the in-control ARL value of an EWMA chart. We support the use of the bootstrap-based design approach of Jones and Steiner (2012) and Gandy and Kvaløy (2013), which was recently proposed for controlling the probability of the in-control ARL being at least a specified value. Our results show that adjusting the EWMA control limits accordingly can result in a highly skewed in-control ARL distribution. However, such increases in the in-control ARL did not have much of an effect on the out-of-control performance of the chart. In our opinion, this design approach is very promising and should be considered while evaluating and comparing control charts. Controlling a percentile of the in-control ARL distribution can provide satisfactory chart performance among a wide range of practitioners.

We found that, if one considers the necessary amount of data for stable performance as determined by the SDARL, fewer observations are required for designing an EWMA chart than a Shewhart chart. Thus, the EWMA chart, with a small smoothing constant, has an advantage over the Shewhart chart, which would require more Phase I data to achieve similar stability in terms of ARL performance across samples. Additionally, based on the bootstrap design procedure, we found that the EWMA chart is much preferred to the Shewhart chart because, with the former, one can simultaneously avoid too frequent false alarms and detect out-of-control sustained shifts more quickly.

Several competing process standard deviation estimators were used as well in assessing the chart performance. Among unbiased estimators, Jones et al. (2001) recommended the use of the estimator $\hat{\sigma}_3 = S_{\text{pooled}}/c_4(v+1)$, and our results agree with this recommendation. However, including the biased estimators in the comparison, we find it preferable to use the estimator $\hat{\sigma}_4 = c_4(v+1)S_{\text{pooled}}$, especially when only a small amount of Phase I data is available. Agreeing with Mahmoud et al. (2010), the range-based estimator was found to be the least efficient compared with the other estimators, and we also recommend against its use.

Appendix A: Calculating the AARL and SDARL for the EWMA Chart with Estimated Parameters Using the Markov Chain Approach

In our article, the EWMA chart is evaluated using the in-control AARL and SDARL metrics. The performance metrics were calculated using the Markov chain approach. Let h be the control limits given in Equation (6), t be the number of the subintervals between the upper and lower control limits (namely, the number of transient states), and w be the width of each subinterval defined as $w = 2h/t$. Saleh et al. (2013, Appendix B) derived the transition probabilities $p_{\ell j}$, $\ell = 1, 2, \ldots, t$ and $j = 1, 2, \ldots, t$, for the EWMA chart when process parameters are estimated. The probability $p_{\ell j}$ refers to the probability of moving from the transient state ℓ to the transient state j. They calculated $p_{\ell j}$ using

$$p_{\ell j} = \varphi\left(Q\left\{\frac{S_j + w/2 - (1-\lambda)S_\ell}{\lambda}\right\} - \gamma + \frac{Z}{\sqrt{m}}\middle|\hat{\mu}_0, \hat{\sigma}_0\right) - \varphi\left(Q\left\{\frac{S_j - w/2 - (1-\lambda)S_\ell}{\lambda}\right\} - \gamma + \frac{Z}{\sqrt{m}}\middle|\hat{\mu}_0, \hat{\sigma}_0\right),$$

where $\varphi(\cdot)$ is the cumulative standard normal distribution function, the quantities Q and Z are defined in Equation (5), and $S_{(\cdot)}$ represents the $(\cdot)$th interval midpoint. We define $\mathbf{R}$ to be a $t \times t$ matrix consisting of the probabilities of moving from one transient state to another such that $\mathbf{R} = [p_{\ell j}]$, and $\mathbf{u}$ to be a $t \times 1$ vector of ones. According to Markov chain approach, the ARL vector is computed as

$$\mathbf{ARL} = (\mathbf{I} - \mathbf{R})^{-1}\mathbf{u}, \tag{A.1}$$

where $\mathbf{I}$ is the identity matrix of dimension $t \times t$. Here, $\mathbf{ARL}$ is a $(t \times 1)$ vector containing the ARLs corresponding to all the possible initial states. We have $Y_0 = 0$. Hence, for an odd value of t, the $(t+1)/2$th element (middle element) corresponds to the ARL satisfying this assumption. The $\mathbf{ARL}$ defined in Equation (A.1) is a function of the random variables $\hat{\mu}_0$ and $\hat{\sigma}_0$, or more generally the random variables Q and Z. Hence, we can write the AARL as

$$\text{AARL} = E(\text{ARL}) = \int_0^\infty \int_{-\infty}^\infty \text{ARL} g_z(z) f_Q(q) dz dq \tag{A.2}$$

and the SDARL as

$$\text{SDARL} = \left[E(\text{ARL}^2) - [E(\text{ARL})]^2\right]^{1/2}, \tag{A.3}$$

where

$$E(\text{ARL}^2) = \int_0^\infty \int_{-\infty}^\infty \text{ARL}^2 g_z(z) f_Q(q) dz dq. \tag{A.4}$$

Here, ARL is the element of the vector $\mathbf{ARL}$ corresponding to the initial state, and the $g_z(z)$ and $f_Q(q)$ are the probability density functions of the random variables Z and Q, respectively. Because the samples are assumed to be i.i.d. normally distributed, the random variables Z and Q are independent. The variable Z follows the standard normal distribution, while Q follows a scaled chi (χ) distribution. Saleh et al. (2013) provided the functional form of the probability density function of Q for each of the standard deviation estimator given in Equation (7). The integrations in Equations (A.2) and (A.4) were approximated using the Gaussian quadrature method. The numerical results were validated using Monte Carlo simulation.

Appendix B: Simplifying the Computations in the Bootstrap Approach

In our article, we design the Shewhart and EWMA control charts using the bootstrap approach. We start here with providing the steps of applying Gandy and Kvaløy's (2013) algorithm, then we list the calculation steps for each of the Shewhart and EWMA chart, along with providing some simplification rules for the Shewhart chart.

Gandy and Kvaløy's Algorithm

The steps of Gandy and Kvaløy's (2013) algorithm for obtaining bootstrap-based control limits can be summarized as follows:

1. Without loss of generality, we let the true unknown in-control distribution P be $N(0, \sqrt{n})$. We generate a Phase I dataset of m samples each of size n from $N(0, \sqrt{n})$ and compute $\hat{\mu}$ and $\hat{\sigma}$. We then compute the quantity $q(\hat{P}, \hat{\theta})$, where $\hat{P} = N(\hat{\mu}, \hat{\sigma})$ is the estimated in-control distribution and $\hat{\theta} = (\hat{\mu}, \hat{\sigma})$ is the estimate of the parameters that are used to run the chart.
2. We then generate $B = 1000$ bootstrap samples from $\hat{P}$ and compute $\hat{\theta}^* = (\hat{\mu}^*, \hat{\sigma}^*)$ for each of the B samples. It is important to note that $\hat{\mu}^*$ and $\hat{\sigma}^*$ are calculated the same way as $\hat{\mu}$ and $\hat{\sigma}$ were calculated.
3. We finally compute the quantities $q(\hat{P}_i^*, \hat{\theta}_i^*)$ and $q(\hat{P}, \hat{\theta}_i^*)$ for $i = 1, 2, 3, \ldots, B$. We obtain the value of $p_{\alpha^*}^*$ as the α percentile of the bootstrap distribution of $q(\hat{P}^*, \hat{\theta}^*) - q(\hat{P}, \hat{\theta}^*)$. The final (adjusted) control limit for the chart is then taken as $q(\hat{P}, \hat{\theta}) - p_{\alpha^*}^*$.

We generated the ARL distribution by repeating the steps 1–3 for a number of times (we used 2000 times). Next, we explain in detail how to compute the different values of q for the Shewhart and EWMA charts.

Gandy and Kvaløy's (2013) approach is based on three limits; $q(\hat{P}, \hat{\theta})$, $q(\hat{P}_i^*, \hat{\theta}_i^*)$, and $q(\hat{P}, \hat{\theta}_i^*)$. The three limits are defined as follows:

(a) The quantity $q(\hat{P}, \hat{\theta})$ represents the value of L that produces the desired in-control ARL when the Phase II data are generated from $\hat{P} = N(\hat{\mu}, \hat{\sigma})$ and the limits are constructed using $\hat{\theta} = (\hat{\mu}, \hat{\sigma})$.

(b) The quantity $q(\hat{P}_i^*, \hat{\theta}_i^*)$, $i = 1, 2, 3, \ldots, B$, represents the value of L that produces the desired in-control ARL when the Phase II data are generated from $\hat{P}_i^* = N(\hat{\mu}_i^*, \hat{\sigma}_i^*)$ and the limits are constructed using $\hat{\theta}_i^* = (\hat{\mu}_i^*, \hat{\sigma}_i^*)$.

(c) The quantity $q(\hat{P}, \hat{\theta}_i^*)$, $i = 1, 2, 3, \ldots, B$, represents the value of L that produces the desired in-control ARL when the Phase II data are generated from $\hat{P} = N(\hat{\mu}, \hat{\sigma})$ and the limits are constructed using $\hat{\theta}_i^* = (\hat{\mu}_i^*, \hat{\sigma}_i^*)$.

Calculation of *q* for the Shewhart Control Chart

Consider the Shewhart control chart where the chart statistic is the sample mean ($\overline{X}$) and the control limits are $\mu \pm L(\sigma/\sqrt{n})$, with L being the control-limit constant chosen to satisfy a specific in-control performance. Finding the quantity $q(\hat{P}, \hat{\theta})$ implies finding L such that

$$P(\hat{\mu} - L\hat{\sigma}/\sqrt{n} < \overline{X} < \hat{\mu} + L\hat{\sigma}/\sqrt{n}) = 1 - \alpha,$$

where α is the false-alarm probability and $\overline{X} \sim N(\hat{\mu}, \hat{\sigma}/\sqrt{n})$. This can be simplified to

$$P\left(\left|\frac{\overline{X} - \hat{\mu}}{\hat{\sigma}/\sqrt{n}}\right| > L\right) = P(|Z| > L) = \alpha.$$

This follows that $L = Z_{1-\alpha/2}$ or equivalently $q(\hat{P}, \hat{\theta}) = Z_{1-\alpha/2}$. Similarly, $q(\hat{P}_i^*, \hat{\theta}_i^*) = L = Z_{1-\alpha/2}$, $i = 1, 2, 3, \ldots, B$. For Shewhart charts, α is the reciprocal of the in-control ARL. Thus, in our study, $\alpha = 0.005$ and thus $Z_{1-\alpha/2} = 2.807$.

For the quantity $q(\hat{P}, \hat{\theta}_i^*)$, we need to find L such that

$$P(\hat{\mu}_i^* - L\hat{\sigma}_i^*/\sqrt{n} < \overline{X} < \hat{\mu}_i^* + L\hat{\sigma}_i^*/\sqrt{n}) = 1 - \alpha,$$

where $\overline{X} \sim N(\hat{\mu}, \hat{\sigma}/\sqrt{n})$ or equivalently

$$P\left(Z < \frac{\hat{\mu}_i^* + L\hat{\sigma}_i^*/\sqrt{n} - \hat{\mu}}{\hat{\sigma}/\sqrt{n}}\right) - P\left(Z < \frac{\hat{\mu}_i^* - L\hat{\sigma}_i^*/\sqrt{n} - \hat{\mu}}{\hat{\sigma}/\sqrt{n}}\right) = 1 - \alpha. \tag{B.1}$$

In such a case, a search algorithm is required for finding the value of L that satisfies Equation (B.1). The search algorithm could be of a binary search type or any other trial and error type. However, these methods may take a long time to produce the results. Thus we provide a short and a quick method for obtaining the value of L.

Our studies showed that L is always bounded between two values. Refer to the quantity in Equation

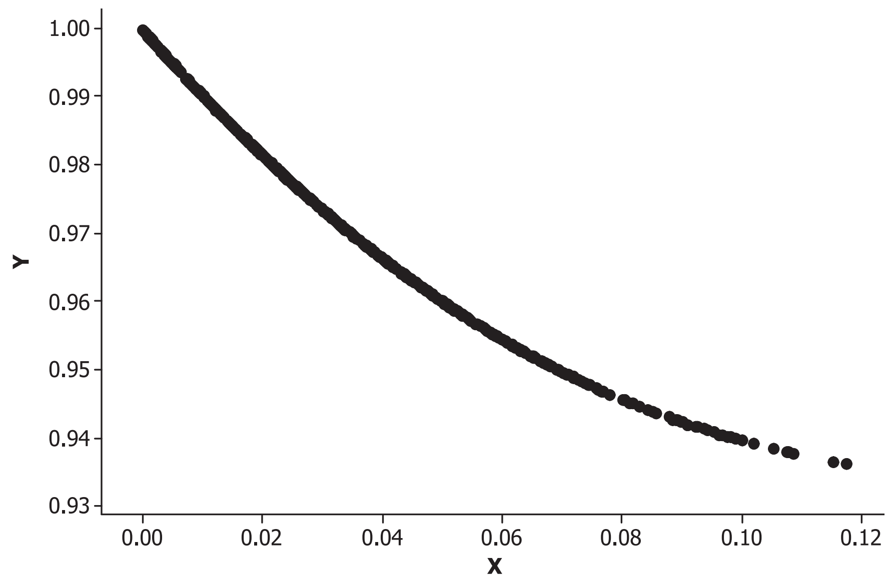

FIGURE B.1. Scatter Plot for Y and X Obtained Using Different Combinations of ($\hat{\mu}$, $\hat{\sigma}$, $\hat{\mu}^*$, $\hat{\sigma}^*$).

(B.1) as $P(Z < b_2) - P(Z < b_1) = 1 - \alpha$. It can be deduced that, if $\hat{\mu}^* \geq \hat{\mu}$, then necessarily $b_1 \geq Z_{\alpha/2}$, which implies that $L \leq (\hat{\sigma}/\hat{\sigma}^*)Z_{1-\alpha/2} + [\sqrt{n}(\hat{\mu}^* - \hat{\mu})/\hat{\sigma}^*]$. Also, if $\hat{\mu}^* \leq \hat{\mu}$, then necessarily $b_2 \leq Z_{1-\alpha/2}$, which implies that $L \leq (\hat{\sigma}/\hat{\sigma}^*)Z_{1-\alpha/2} + [\sqrt{n}(\hat{\mu} - \hat{\mu}^*)/\hat{\sigma}^*]$. Hence, an upper bound for L can be defined as $(\hat{\sigma}/\hat{\sigma}^*)Z_{1-\alpha/2} + (\sqrt{n}|\hat{\mu} - \hat{\mu}^*|/\hat{\sigma}^*)$, where $|\cdot|$ denotes the absolute value. Additionally, $b_2 - b_1 > 2Z_{1-\alpha/2}$ because the symmetric interval is the shortest interval for a standard normal density containing a given probability. Thus, substituting with the expressions of b_1 and b_2 and solving for L provides that $L \geq (\hat{\sigma}/\hat{\sigma}^*)Z_{1-\alpha/2}$. Consequently, we can say that $L \in (L_1, L_2)$, where

$$L_1 = \frac{\hat{\sigma}}{\hat{\sigma}^*} Z_{1-\alpha/2} \quad \text{and} \quad L_2 = L_1 + \Delta, \qquad \text{(B.2)}$$

where $\Delta = \sqrt{n}(|\hat{\mu} - \hat{\mu}^*|/\hat{\sigma}^*)$.

An important finding is that a very strong relationship was found between the variables $Y = L/L_2$ and $X = \Delta/L_2$. Figure B.1 is a scatterplot of 500 observations of Y and X (obtained using different combinations of $\hat{\mu}$, $\hat{\sigma}$, $\hat{\mu}^*$, $\hat{\sigma}^*$ and a search algorithm for L).

Using a curve fitting technique, we found that a cubic regression model between Y and X can fit this relationship almost perfectly. The best fit was found to be related to the values of Δ and x. That is, if $\Delta < 0.626$ and $x < 0.08$, then we use

$$y = 0.99985 - 1.01234x + 4.48698x^2 - 3.97727x^3.$$

If $\Delta < 0.626$ and $x \geq 0.08$, then we use

$$y = 1.00178 - 1.09381x + 5.60924x^2 - 8.95849x^3.$$

Given the satisfaction of these conditions and the availability of $\hat{\mu}$, $\hat{\sigma}$, $\hat{\mu}^*$, and $\hat{\sigma}^*$, we can find the values of L_1, L_2 and X and substitute into the fitted model to obtain the value of $L = yL_2$. The fitted models do not provide accurate results for the case of $\Delta \geq 0.626$, but this case is uncommon. Consequently, if $\Delta \geq 0.626$, a usual trial-and-error search algorithm is used to find the value of L that satisfies Equation (B.1).

So, for the Shewhart chart, we can summarize step 3 of our algorithm as follows. Calculate Δ and x. If $\Delta < 0.626$, use the appropriate fitted regression model based on the value of x to find $q(\hat{P}, \hat{\theta}_i^*)$. Otherwise, we apply a search algorithm. An advantage for having $q(\hat{P}, \hat{\theta}) = q(\hat{P}^*, \hat{\theta}^*) = Z_{1-\alpha/2}$ is that adjusted limit can be reduced to be only taking the $100(1-\alpha^*)\%$ percentile of $q(\hat{P}, \hat{\theta}_i^*)$, $i = 1, 2, 3, \ldots, B$.

Calculation of *q* for the EWMA Control Chart

For the EWMA chart, finding the quantity $q(\hat{P}, \hat{\theta})$ is similar to the case of finding $q(P, \theta)$; i.e., the value of L that produces the desired in-control ARL when the in-control process parameters are known. This is because the in-control distribution is defined with the same estimated parameters $(\hat{\theta})$ used in building the control chart limits. Hence, it follows that $q(\hat{P}, \hat{\theta})$

is equal to 2.454 for $\lambda = 0.1$. Similarly, $q(\hat{P}_i^*, \hat{\theta}_i^*) = 2.454$ for $i = 1, 2, 3, \ldots, B$.

However, this result can't be extended in finding the quantity $q(\hat{P}, \hat{\theta}_i^*)$ because the estimates of the in-control distribution differ from that corresponding to the control limits. Hence, a search algorithm is required for finding the value of L satisfying an in-control ARL of 200. The search algorithm would be of a trial-and-error type and should be associated with a validation technique (e.g., a Markov chain code) in each and every iteration. It should be noticed that, if the Markov chain approach described in Appendix A is used in the validation process, then the standardized sample mean $\hat{W}$ should be based on the quantities $Q = \hat{\sigma}^*/\hat{\sigma}$, $\nu_i = \sqrt{n}(\overline{X}_i - (\hat{\mu}+\delta))/\hat{\sigma}$, $\gamma = \sqrt{n}\delta/\hat{\sigma}$, and $Z = \sqrt{mn}(\hat{\mu}^* - \hat{\mu})/\hat{\sigma}$.

This method can take a long time to produce results. A similar simplifying rule to that of the Shewhart chart can probably be used for the EWMA chart, but further investigation is needed to establish this. As mentioned in the Shewhart chart case, we have the equality of $q(\hat{P}, \hat{\theta})$ and $q(\hat{P}^*, \hat{\theta}^*)$ and we let the adjusted limit be the $100(1-\alpha^*)\%$ percentile of $q(\hat{P}, \hat{\theta}_i^*)$, $i = 1, 2, \ldots, B$.

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