A Robust Estimator for Location in Phase I Based on an EWMA Chart

INEZ M. ZWETSLOOT, MARIT SCHOONHOVEN, and RONALD J. M. M. DOES

University of Amsterdam, Plantage Muidergracht 12, 1018 TV Amsterdam, The Netherlands

In practice, a control chart for process monitoring (Phase II) is based on parameters estimated from data collected on the process characteristic under study (Phase I). The Phase I data could contain unacceptable data, which in turn could affect the monitoring. In this study, we consider various estimation methods that are potentially relevant within the parameter estimation process. The quality of the Phase I study is evaluated in terms of the precision of the resulting estimates as well as the effectiveness of the exploratory data analysis, where 'effectiveness' is measured by the proportion of observations that are correctly identified as unacceptable. Moreover, we study the impact of the Phase I estimation method on the performance of the EWMA control chart in Phase II.

Key Words: Changepoint; Control Chart; Estimation; Monitoring; Outliers; Statistical Process Control.

1. Introduction

CONTROL charts are designed to monitor a process characteristic through time. The original control chart was developed by Shewhart (1926, 1931). To increase the sensitivity of the Shewhart control chart, additional decision rules may be appropriate (Western Electric Company (1956)). The first memory-type control chart was introduced by Page (1954) and is known as the cumulative sum (CUSUM) control chart. Another commonly used memory-type control chart is the exponentially weighted moving average (EWMA) control chart developed by Roberts (1959). Both control charts are able to detect small sustained changes in the process mean.

Hunter (1986) provided a useful analysis of the degree in which the Shewhart, CUSUM, and EWMA control charts use the history of the data to detect a change in the process mean. He pointed out that the Shewhart control chart only uses the current observation and therefore has no memory while, in contrast, the CUSUM control chart uses all of the history, paying equal attention to all past observations and the current observation. This is an oversimplification because, in fact, the CUSUM chart uses a decision rule whereby some of the past observations can become irrelevant. The third control chart variant, the EWMA control chart, gives less weight to data as they get older. The weight given to the current observation relative to earlier observations can be chosen by selecting a smoothing parameter between 0 and 1: a value of 1 means that all of the weight is assigned to the current observation and no weight to previous observations, equivalent to the Shewhart control chart, while a value of almost 0 results in a control chart with a long memory. We shall use these specific properties of the EWMA control chart in this paper.

It is generally accepted in the literature that a control chart is implemented in two phases: Phase I, to define the stable state of the process characteristic and to estimate its distributional parameters; and Phase II, to monitor the process (Vining (2009), Chakraborti et al. (2009)). In a survey of Phase I analysis, Jones-Farmer et al. (2014) review the major issues and developments in Phase I analysis. One of the issues is the possibility of unacceptable data in Phase I, because these observations could have an

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Ms. Zwetsloot is a Consultant in Statistics at IBIS UvA and PhD student. Her email address is i.m.zwetsloot@uva.nl.

Dr. Schoonhoven is a Senior Consultant in Statistics at IBIS UvA and Associate Professor at the University of Amsterdam. Her email address is m.schoonhoven@uva.nl.

Dr. Does is Professor of Industrial Statistics at the University of Amsterdam, Managing Director of IBIS UvA, and Director of the Institute of Executive Programmes of the University of Amsterdam. He is a Fellow of ASQ. His email address is r.j.m.m.does@uva.nl.

impact on Phase II performance. The present article studies several location estimation methods. We assess their effectiveness in Phase I and their impact on Phase II EWMA control chart performance given that Phase I could contain unacceptable data.

Let Y_{it}, with $i = 1, 2, \ldots, n$ and $t = 1, 2, \ldots$, denote the Phase II data, assumed to be independently and identically $N(\mu, \sigma^2)$ distributed when the process is on target. In Phase II, we wish to monitor the mean of the process characteristic and detect changes from its target as soon as possible. Throughout this article, we assume that the target is equal to μ.

The EWMA control chart uses a weighted average of current and past observations as a monitoring statistic. At sampling stage t, the EWMA statistic is defined as

$$Z_t = (1 - \lambda)Z_{t-1} + \lambda \overline{Y}_t,$$

with $\overline{Y}_t$ the mean of sample t, $Z_0 = \hat{\mu}$ (where $\hat{\mu}$ denotes an estimate of μ), and the smoothing constant λ satisfying $0 < \lambda \leq 1$. A smaller λ implies a heavier reliance on the past and therefore a quicker detection of small, sustained shifts (cf. Lucas and Saccucci (1990)). Under the assumption of independently, identically, and normally distributed Y_{it}, the mean and standard deviation of Z_t are equal to $\hat{\mu}$ and $\hat{\sigma}_{Z_t}$, the latter estimated by

$$\hat{\sigma}_{Z_t} = \frac{\hat{\sigma}}{\sqrt{n}} \sqrt{\frac{\lambda}{2 - \lambda}[1 - (1 - \lambda)^{2t}]}.$$

Then the upper control limit (UCL) and lower control limit (LCL) can be determined by

$$\widehat{\text{UCL}}_t = \hat{\mu} + L\hat{\sigma}_{Z_t} \quad \text{and} \quad \widehat{\text{LCL}}_t = \hat{\mu} - L\hat{\sigma}_{Z_t},$$

where L is a positive coefficient which, together with λ, determines the performance of the EWMA control chart when the process is stable. When Z_t falls above $\widehat{\text{UCL}}_t$ or below $\widehat{\text{LCL}}_t$, the control chart signals unstable performance.

Jones et al. (2001) studied the EWMA control chart with estimated parameters and Saleh et al. (2014) studied the Shewhart control chart with estimated parameters. Both showed that, in order to have on average the performance of a control chart with known parameters, many more samples are needed than the 100 samples of size 5 recommended for the Shewhart control chart (see Quesenberry (1993)). For example, for EWMA control charts with $\lambda = 0.5$, 200 samples of size 5 are required. When $\lambda = 0.1$, 400 samples of size 5 are needed to achieve a performance similar to that of a control chart with

known parameters. As argued by Jones (2002), these sample sizes are often unavailable in practice. To take account of the additional uncertainty, they develop an alternative design procedure that provides corrected values for L. Jones (2002) applied this new method in an example, and uses $k = 50$ samples of size $n = 5$ to estimate μ and σ.

In our paper, we study the estimation of the distributional parameters when limited Phase I data are available and these data may contain contaminations, which is often the case in practice. Jensen et al. (2006) noted that "it seems appropriate to use estimators that will be robust to outliers, step changes, and other data anomalies in Phase I." Numerous robust parameter estimates have been discussed in the literature (see Rocke (1989), Tatum (1997), among others). Another possibility is the use of Phase I Shewhart charts (see Chakraborti et al. (2009), Schoonhoven et al. (2011a)). Nazir et al. (2013) studied robust point location estimators in the context of Phase II monitoring with a CUSUM control chart. These methods are generally very useful where one occasional sample or observation is not acceptable. However, some processes are subject to sustained shifts. We therefore include a method particularly suited to detect sustained shifts, namely the changepoint method (for an overview see Amiri and Allahyari (2012)), and a new estimation method that makes use of the EWMA chart in Phase I. The methods are evaluated in terms of their impact on Phase II EWMA control chart performance, as well as in terms of the effectiveness of the Phase I study itself. The Phase I study should be used as an exploratory data analysis tool and it is therefore important that unacceptable observations are identified.

The article is organized as follows. The estimation methods outlined above are presented in Section 2. Both the set-up and the findings of the Phase I study are presented in Section 3. Section 4 describes the results for the Phase II context and, in Section 5, we summarize our conclusions and recommendations.

2. Phase I Estimation Methods

In this section, we describe various estimation methods that can be used within Phase I, the exploratory data analysis stage. We consider point estimators and—especially useful for the detection of sustained shifts—a changepoint method. We also present estimation methods used in classical statistical process control (SPC) in Phase I.

Let X_{it}, with $i = 1, 2, \ldots, n$ and $t = 1, 2, \ldots, k$,

represent the Phase I observations, and assume X_{it} to be independently and identically $N(\mu, \sigma^2)$ distributed with mean μ and standard deviation σ if the process is stable. Throughout our study, we set $k = 50$ and $n = 5, 10$.

2.1. Point Estimators

We consider two point estimators with different characteristics: the grand sample average $\overline{\overline{X}}$, known as the most efficient estimator for the location under uncontaminated normality, and the median of the sample averages $M(\overline{X})$, known for its robustness against data anomalies (see Janacek and Meikle (1997)). The grand sample average is defined by

$$\overline{\overline{X}} = \frac{1}{k}\sum_{t=1}^{k}\overline{X}_t = \frac{1}{k}\sum_{t=1}^{k}\left(\frac{1}{n}\sum_{i=1}^{n}X_{it}\right),$$

and the median of the sample averages by

$$M(\overline{X}) = \text{median}(\overline{X}_1, \ldots, \overline{X}_k).$$

2.2. Changepoint Methods

The changepoint method compares the log likelihood of all observations, under the assumption that all observations are in control, with the log likelihood of the observations if a step change has occurred. Sullivan and Woodall (1996) proposed a changepoint method for exploratory data analysis and showed that this method outperforms the Shewhart chart in detecting sustained shifts. We include their changepoint method in our analysis.

Consider the set $S_0 = (1, 2, \ldots, k)$, which we can split into sets $S_1 = (1, 2, \ldots, \tau)$ and $S_2 = (\tau + 1, \tau + 2, \ldots, k)$, with $1 \leq \tau \leq k-1$. Denoting the number of elements in set S_j by s_j, we have $s_0 = k$, $s_1 = \tau$, and $s_2 = k - \tau$. The maximum-likelihood estimators of the mean and variance of the observations in samples $t \in S_j$ are given by

$$\widetilde{\mu}_j = \frac{1}{n * s_j}\sum_{t \in S_j}\sum_{i=1}^{n}X_{it}$$

and

$$\widetilde{\sigma}_j^2 = \frac{1}{n * s_j}\sum_{t \in S_j}\sum_{i=1}^{n}(X_{it} - \widetilde{\mu}_j)^2 \qquad \text{for } j = 0, 1, 2.$$

To test for the existence of a step change after sample τ, we compute the likelihood-ratio statistic as $\text{LRT}(\tau) = nk\ln[\widehat{\sigma}_0^2] - n\tau\ln[\widehat{\sigma}_1^2] - n(k - \tau)\ln[\widehat{\sigma}_2^2]$ (see Sullivan and Woodall (1996)).

Sullivan and Woodall (1996) constructed a chart by plotting the $\text{LRT}(\tau)$ statistic against τ, with an

out-of-control signal occurring if a value exceeds the UCL. However, if τ is close to 1 or $k - 1$, the expected value of $\text{LRT}(\tau)$ is larger than for an intermediate τ. We standardize $\text{LRT}(\tau)$ by its expected value $E(\text{LRT}(\tau))$. The standardized test statistic is denoted by $\text{LRT}'(\tau)$ and is compared with a constant upper control limit UCL_{CP}. The expected values $E(\text{LRT}(\tau))$ are determined through 100,000 simulations, as is also done in Sullivan and Woodall (1996), and are presented in the appendix.

A sustained shift in the Phase I data set is detected if $\text{LRT}'(\tau)$ exceeds UCL_{CP}. Every out-of-control signal indicates a probable sustained shift in the process. When multiple signals are given, we set the estimated changepoint ($\hat{\tau}$) equal to the τ for which $\text{LRT}'(\tau)$ is maximal. If there is no out-of-control signal, we set $\hat{\tau} = k$.

To determine UCL_{CP}, we set the overall in-control false alarm probability equal to 0.01. Using 100,000 simulations, we find that $\text{UCL}_{\text{CP}} = 5.75$ for $k = 50$ and for both $n = 5$ and $n = 10$.

When the changepoint $\hat{\tau}$ is estimated, we can determine which samples are out of control. In practice, knowledge of the process would be used to determine whether the data before or after the estimated $\hat{\tau}$ are in control. In order to prevent the deletion of a large proportion of clean observations from the Phase I dataset (this problem could occur in our simulation if there is a false alarm at the beginning of the Phase I dataset), we use the following decision rule: 'the majority of the samples represent the in-control process'. This implies that, if $\hat{\tau} \leq k/2$, we delete samples 1 up to $\hat{\tau}$ from Phase I. If $\hat{\tau} > k/2$, we delete samples $\hat{\tau}+1$ up to k from Phase I. The remaining samples are used to compute the grand sample mean, yielding an estimate of μ based on changepoint analysis, which we denote by CP. We believe this is an appropriate decision rule, as practitioners can investigate which sequence before or after the shift is acceptable. Moreover, it ensures a more suitable comparison with the other Phase I methods considered in our study. This changepoint method is designed to detect a single changepoint $\hat{\tau}$ and is at a disadvantage if multiple step changes occur in Phase I. Alternative changepoint methods can be designed based on recursive testing for step changes.

2.3. SPC-Based Estimation Methods

Shewhart control charts are known for their detection of scattered outliers but not for their effectiveness when Phase I contains sustained shifts. Other

methods should then be selected, such as change-point methods (see Sullivan and Woodall (1996)) and EWMA charts (see Hunter (1986)). In this section, we consider the use of the EWMA chart in Phase I.

The proposed Phase I EWMA screening estimators consist of the following steps. First, initial estimates of the location and standard deviation, $\widehat{\mu}_I$ and $\widehat{\sigma}_I$, are obtained. The choice of these estimators is discussed below. Next, a Phase I EWMA chart is set up with EWMA statistic $Z_t = (1 - \lambda_I)Z_{t-1} + \lambda_I \overline{X}_t$ and control limits

$$\widehat{\text{UCL}}_t / \widehat{\text{LCL}}_t = \widehat{\mu}_I \pm L_I \frac{\widehat{\sigma}_I}{\sqrt{n}} \sqrt{\frac{\lambda_I}{2 - \lambda_I}[1 - (1 - \lambda_I)^{2t}]},$$

with $Z_0 = \widehat{\mu}_I$. Note that the subscript 'I' denotes that the parameter is associated with Phase I charting. If Z_t falls above $\widehat{\text{UCL}}_t$ or below $\widehat{\text{LCL}}_t$, the corresponding sample is identified as unacceptable and deleted from Phase I. Then we can use an efficient estimator of μ, yielding an estimate that is efficient as well as robust to various patterns of outliers. The final location estimate is given by the mean of the remaining sample means,

$$S\widehat{\mu}_{\lambda_I} = \frac{1}{k^*} \sum_{t=1}^{k} \overline{X}_t \times I(\widehat{\text{LCL}}_t < Z_t < \widehat{\text{UCL}}_t).$$

Here, $I(.)$ is the indicator function, k^* denotes the number of remaining samples, and S indicates that the estimator is based on Phase I charting.

The choice of $\widehat{\mu}_I$ is an important one: an efficient estimator could improve the performance of the Phase I chart under stability but inflate the Phase I control limits when disturbances are present. On the other hand, a robust estimator could result in nonoptimal performance under stability, but in robust Phase I control limits when disturbances are present. Our study evaluates the impact of the most efficient estimator for the location, the grand sample average ($\overline{\overline{X}}$), and a robust estimator based on the median of the sample means ($M(\overline{X})$).

We use just one method to derive $\widehat{\sigma}_I$ so that any differences in performance are due to differences in the estimation of μ. We use an estimator of σ that is known for its robustness, namely, a variant of the biweight A estimator proposed by Tatum (1997). This estimator weights residuals; residuals are computed as the difference between the observations and the sample medians. Large residuals are given less weight than smaller residuals, which ensures that outliers have less impact on the estimate of σ. The estimation procedure is described by Tatum (1997) and is implemented as set out in Schoonhoven et al. (2011b), with normalizing constants 1.068 for $n = 5$ and 0.962 for $n = 10$.

As far as the choice of λ_I is concerned, we take a range of values in order to study the impact of this parameter. Small values for λ_I enable detection of sustained shifts, while larger values of λ_I enable detection of scattered outliers. To assess this trade-off, we set λ_I equal to 0.2, 0.6, and 1. When $\lambda_I = 1$, we obtain the Shewhart chart. To obtain values for L_I, we set the false-alarm rate at 1%, thereby following Chakraborti et al. (2009).

Table 1 gives an overview of the Phase I estimators considered and the corresponding values of L_I (obtained through 100,000 Monte Carlo simulations).

3. Phase I Study

In this section, we evaluate the effectiveness of Phase I studies that use the methods presented in Table 1. One of the requirements of Phase I is to de-

TABLE 1. Phase I Location Estimators

Estimator	Description	L_I	
		$n = 5$	$n = 10$
$\overline{\overline{X}}$	The grand sample average	n.a.	n.a.
$M(\overline{X})$	Median of the sample averages	n.a.	n.a.
CP	Changepoint estimator	n.a.	n.a.
$S\overline{\overline{X}}_{0.6}$	Phase I screening estimator with $\widehat{\mu}_I = \overline{\overline{X}}$ and $\lambda_I = 0.6$	2.540	2.525
$SM(\overline{X})_{0.2}$	Phase I screening estimator with $\widehat{\mu}_I = M(\overline{X})$ and $\lambda_I = 0.2$	2.540	2.525
$SM(\overline{X})_{0.6}$	Phase I screening estimator with $\widehat{\mu}_I = M(\overline{X})$ and $\lambda_I = 0.6$	2.610	2.592
$SM(\overline{X})_1$	Phase I screening estimator with $\widehat{\mu}_I = M(\overline{X})$ and $\lambda_I = 1$	2.617	2.600

liver accurate parameter estimates. We assess the estimation precision of the proposed methods in terms of the mean squared error (MSE). In addition, the Phase I analysis is used as a tool for exploratory data analysis, allowing us to examine the data and learn from out-of-control observations. Section 3.1 describes the data scenarios considered in Phase I, and Sections 3.2 and 3.3 present the Phase I results in terms of the MSE and the percentage of identified out-of-control observations, respectively.

3.1. Data Scenarios

Recall that the stable, in-control, Phase I data are assumed to be $N(\mu, \sigma^2)$ distributed. One of the requirements of Phase I is to deliver an accurate parameter estimate of μ, even if Phase I contains contaminated observations. In our paper, we assume that contaminated observations come from a shifted normal distribution $N(\mu + \delta_I \sigma, \sigma^2)$, with δ_I a constant. The Shewhart control chart is particularly useful to detect single, relatively large disturbances in the data or *scattered* special causes. These disturbances are transient in that they affect single samples. But, in some situations, disturbances might be *sustained* in that they last for at least a few consecutive samples beyond their first appearance. This is where EWMA charts play a role. In the study, we consider both scattered and sustained special causes. The scenarios are described below, where we set the parameters for the stable state at $\mu = 0$ and $\sigma = 1$, without loss of generality.

1. A model for *localized* shifts in which all observations in a sample have a 90% probability of being drawn from the $N(0, 1)$ distribution and a 10% probability of being drawn from the $N(\delta_I, 1)$ distribution.

2. A model for *diffuse* shifts in which each observation, irrespective of the sample to which it belongs, has a 90% probability of being drawn from the $N(0, 1)$ distribution and a 10% probability of being drawn from the $N(\delta_I, 1)$ distribution.

3. A model for a *single step shift* in which the first 45 samples are drawn from the $N(0, 1)$ distribution and the last 5 samples are drawn from the $N(\delta_I, 1)$ distribution.

4. A model for *multiple step shifts* in which, at each time point, the sample has a probability p of being the first of five consecutive samples drawn from the $N(\delta_I, 1)$ distribution. After any such step shift, each sample again has a probability p of being the start of another step shift.

Phase I consists of 50 samples. If sample 48 shifts, then only 3 samples (48, 49, and 50) are drawn from the $N(\delta_I, 1)$ distribution, instead of five. To maintain the 10% expected contamination rate of models 1–3, we set $p = 0.023$.

The performance of the proposed estimators is evaluated for scenarios where $\delta_I = 0, 0.2, 0.4, \ldots, 2$.

3.2. Estimation Accuracy

In order to determine the accuracy of the proposed location estimators, we determine the MSE for each estimation method under the scenarios proposed in the previous subsection. The MSE is calculated as

$$\text{MSE} = \frac{1}{R} \sum_{r=1}^{R} \left(\frac{\hat{\mu}^r - \mu}{\sigma} \right)^2 = \frac{1}{R} \sum_{r=1}^{R} (\widehat{\mu}^r)^2, \quad (1)$$

where $\hat{\mu}^r$ is the value of an estimator in the rth simulation run and R is the total number of simulations in the Monte Carlo study. We set $R = 200,000$ with a relative simulation error—the standard error of the estimated MSEs expressed as a percentage of the MSE—which never exceeds 0.5%. The MSE results are presented in Figures 1 through 4.

The y-intercept shows the MSE of the estimation methods when the data are stable ($\delta_I = 0$). The estimator $\overline{\overline{X}}$ shows the lowest MSE, as expected. The other estimators show only slightly larger MSE levels, except for $M(\overline{X})$, which is less efficient under stability.

Next, we study the situation when contaminations are present in Phase I ($\delta_I > 0$). We see that the traditional point estimator $\overline{\overline{X}}$ is most sensitive to all data disturbances considered. The estimator $M(\overline{X})$ as well as the screening estimators are rather robust in the scenarios where the mean of an entire sample has shifted, namely localized, single and multiple step shifts (see Figures 1, 3, and 4), but not when diffuse disturbances are present (see Figure 2). The reason is that these estimators trim the sample means rather than extreme observations within a sample. The estimator CP has the lowest MSE when there are single step shifts (Figure 3), but its performance in other situations is far worse than that of the alternative estimators.

As regards the choice of λ_I for the screening methods, we had expected that the Phase I Shewhart chart, $SM(\overline{X})_1$, would perform best for localized disturbances as the Shewhart chart is well known for its detection of single (extreme) disturbances. However,
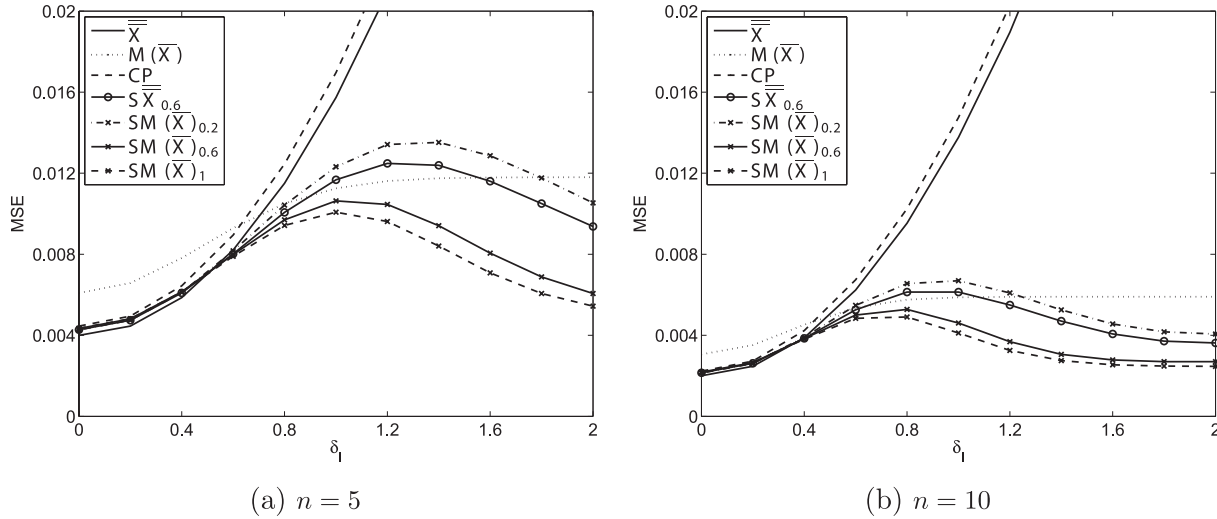
FIGURE 1. MSE of Location Estimators when Localized Shifts Are Present in Phase I for $k = 50$ Samples of Size n. (a) $n = 5$; (b) $n = 10$.

from Figure 1, we can see that the Phase I Shewhart chart is only slightly superior to the Phase I EWMA chart with $\lambda_I = 0.6$ $(SM(\overline{X})_{0.6})$. Moreover, $SM(\overline{X})_{0.6}$ performs better when there are single or multiple step shifts. Note that an EWMA chart with a lower λ_I, for example $SM(\overline{X})_{0.2}$, does not perform as well for short and large sample disturbances: a lower value of λ_I is more suitable for smaller sustained shifts. Because the disturbances in applications can be scattered as well as sustained, we recommend in Phase I the use of an EWMA chart with

$\lambda_I = 0.6$ or a similar intermediate value, rather than a Shewhart chart.

As for the choice of $\widehat{\mu}_I$, i.e., whether we use $SM(\overline{X})_{0.6}$ or $S\overline{\overline{X}}_{0.6}$, it is worth noting that the method based on the robust estimator $M(\overline{X})$ for the Phase I chart is as efficient under stable data ($\delta_I = 0$) as the chart based on $\overline{\overline{X}}$. This becomes clear when we realize that both charts use the efficient estimator $\overline{\overline{X}}$ to determine the mean after screening. We can conclude that it does not matter for efficiency of

FIGURE 2. MSE of Location Estimators when Diffuse Shifts Are Present in Phase I for $k = 50$ Samples of Size n. (a) $n = 5$; (b) $n = 10$.
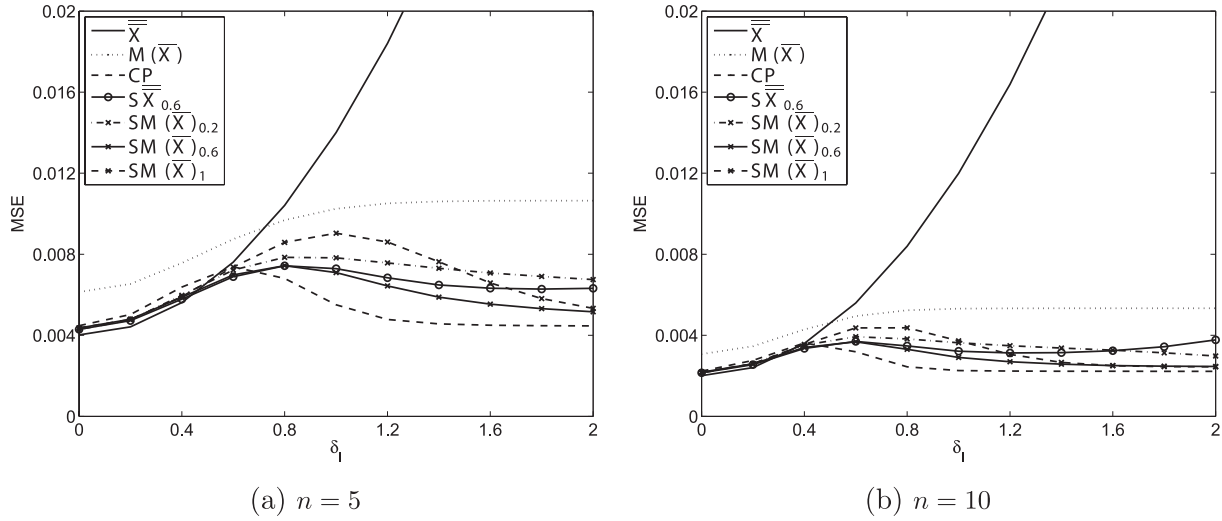
(a) $n = 5$ (b) $n = 10$

FIGURE 3. MSE of Location Estimators when a Single Step Shift Is Present in Phase I for $k = 50$ Samples of Size n. (a) $n = 5$; (b) $n = 10$.

the final estimate whether a less efficient estimator is used to construct the Phase I chart. The use of a robust estimator like $M(\overline{X})$ for the Phase I chart does pay off, however: when there are large multiple step shifts (Figure 4 for $\delta_I > 0.8$), we see that the performance of the Phase I chart based on $\overline{\overline{X}}$ is not good. The higher the value of δ_I, the higher will be the MSE. When a nonrobust estimator is used for the Phase I chart, disturbances might affect the Phase I limits so that the wrong observations are filtered out of the data. As the type of disturbance in Phase I

is not known in advance, we recommend the use of a Phase I chart based on a robust estimator such as $SM(\overline{X})_{0.6}$ rather than a Phase I chart based on an efficient estimator such as $S\overline{\overline{X}}_{0.6}$.

Finally, note that none of the proposed estimation methods perform well when there are diffuse disturbances, i.e., contaminated observations scattered over the entire Phase I data set. Because the estimators screen whole samples, they do not identify these individual scattered outliers and therefore use all ob-
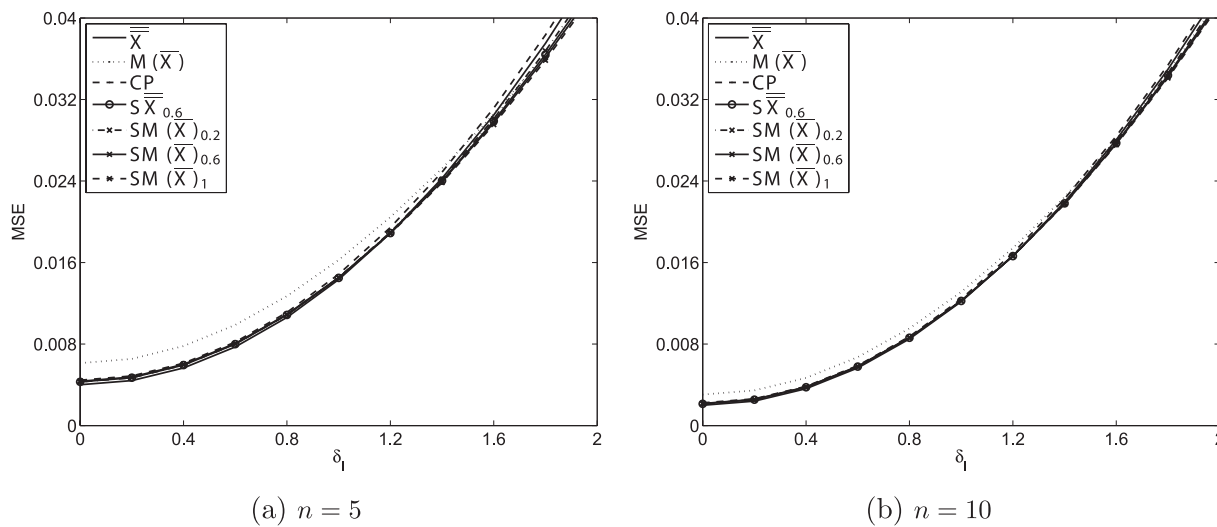

(a) $n = 5$ (b) $n = 10$

FIGURE 4. MSE of Location Estimators when Diffuse Shifts Are Present in Phase I for $k = 50$ Samples of Size n. (a) $n = 5$; (b) $n = 10$.
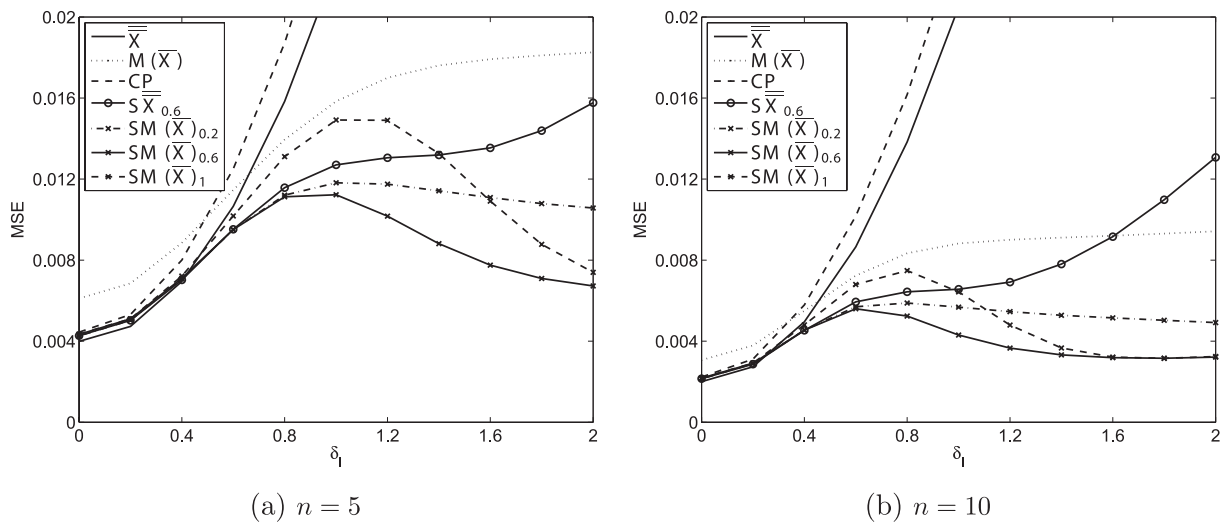
TABLE 2. Maximum Relative Mean-Squared Error (RMSE) and, in Bold, the Estimator with the Lowest Maximum RMSE

Scenario	n	Phase I estimators for μ						
		$\overline{\overline{X}}$	$M(\overline{X})$	CP	$S\overline{\overline{X}}_{0.6}$	$SM(\overline{X})_{0.2}$	$SM(\overline{X})_{0.6}$	$SM(\overline{X})_1$
Localized	5	835.8	116.7	935.6	73.2	93.5	13.9	**8.9**
	10	1908.5	141.3	2125.5	72.3	92.7	13.7	**8.9**
Diffuse	5	**6.4**	52.6	11.2	7.0	6.5	8.4	8.8
	10	**3.4**	53.5	11.5	7.0	6.8	8.6	9.0
Single step	5	887.1	139.9	**13.9**	44.2	61.1	35.6	81.5
	10	1787.7	139.7	**15.3**	69.7	60.5	36.1	79.3
Multiple step	5	1071.4	173.5	1247.0	136.6	58.6	**8.4**	50.9
	10	2153.6	188.8	2458.4	289.8	59.8	**8.7**	49.9
All scenarios combined		2153.6	188.8	2458.4	289.8	93.5	**36.1**	81.5

servations to estimate the location. This means that the estimators are based on a mixture of in-control and out-of-control observations and will be approximately equal to $0.1\delta_I$, yielding a MSE level approximately equal to $0.1\delta_I^2$, as can be observed in Figure 2. We think that the proposed estimators can be augmented with a method that screens for individual outliers and see this as an issue for future research.

To compare the performance of the proposed estimators across the various contamination schemes, we compute the relative mean squared error (RMSE) of the estimators. The RMSE of an estimator, for a specific type of data contamination and severity δ_I, is defined as the percentage increase in the MSE of the estimator relative to the MSE of the estimator with the lowest MSE. For each data scenario and each estimator, we obtain the RSME of the estimator for all levels of δ_I. We present the maximum RSME over δ_I for each estimator in Table 2. The table shows that, in the presence of localized disturbances, the estimator $SM(\overline{X})_1$ has the lowest maximum RMSE (i.e., has an MSE that overall is closest to the optimal MSE for all shift sizes). When there is a single step shift, the changepoint estimator has the lowest RMSE: its MSE is at most 15.3% larger than the optimal estimator for all δ_I. If we consider all disturbance scenarios together (last row in Table 2), we find that the estimator $SM(\overline{X})_{0.6}$ is always within 36.1% of the optimal estimators' MSE, irrespective of the pattern of contaminations.

3.3. Phase I Detection Probability

In practice, the Phase I analysis is also used as a tool for exploratory data analysis. One would like to 'learn from the data'. If such an analysis is to be effective, it must identify unacceptable observations without triggering false alarms for acceptable observations. In this section, we measure the effectiveness of the Phase I study in terms of the true-alarm percentage (TAP) and the false-alarm percentage (FAP). Related measures are presented by Fraker et al. (2008), Chakraborti et al. (2009), and Frisén (2009). The TAP and FAP are calculated as

$$\text{TAP} = \frac{1}{R} \sum_{r=1}^{R} \frac{(\#\text{correct signals})^r}{(\#\text{unacceptable observations})^r} \times 100\%$$

and

(2)

$$\text{FAP} = \frac{1}{R} \sum_{r=1}^{R} \frac{(\#\text{false alarms})^r}{(\#\text{acceptable observations})^r} \times 100\%,$$

(3)

where r denotes the rth simulation run.

Note that the analysis could only be carried out for the proposed estimation methods that have a screening procedure to identify unacceptable samples in Phase I. These are CP, $S(\overline{\overline{X}})_{0.6}$, $SM(\overline{X})_{0.2}$, $SM(\overline{X})_{0.6}$, and $SM(\overline{X})_1$. The results are presented in Table 3.

Some interesting findings on the TAP and FAP are the following:

TABLE 3. True-Alarm Percentage (TAP) and False-Alarm Percentage (FAP)

			TAP				FAP				
				δ_{I}					δ_{I}		
Scenario	n	$\hat{\mu}$	0.4	1	1.6	2	0	0.4	1	1.6	2
Localized	5	CP	1.4	4.1	11.2	17.0	1.1	1.3	2.6	6.7	11.1
		$S\overline{\overline{X}}_{0.6}$	3.3	23.0	61.1	82.2	1.0	1.1	1.5	2.3	3.2
		$SM(\overline{X})_{0.2}$	2.3	15.7	46.7	67.4	1.0	1.2	2.3	5.9	9.7
		$SM(\overline{X})_{0.6}$	3.4	26.2	71.1	90.9	1.0	1.1	1.3	1.8	2.4
		$SM(\overline{X})_1$	3.8	29.8	77.7	94.8	1.0	1.0	1.1	1.1	1.1
	10	CP	1.8	9.0	21.3	26.9	1.1	1.5	4.9	13.1	18.4
		$S\overline{\overline{X}}_{0.6}$	6.5	49.6	91.2	98.6	1.0	1.2	2.0	4.0	6.1
		$SM(\overline{X})_{0.2}$	4.0	36.8	78.5	93.0	1.0	1.4	4.6	12.6	18.7
		$SM(\overline{X})_{0.6}$	6.7	58.3	97.0	99.9	1.0	1.1	1.6	3.1	4.7
		$SM(\overline{X})_1$	7.6	64.9	98.7	100.0	1.0	1.1	1.1	1.1	1.1
Diffuse	5	CP	1.1	1.2	1.4	1.9	1.1	1.1	1.2	1.4	1.9
		$S\overline{\overline{X}}_{0.6}$	1.1	1.4	2.2	2.9	1.0	1.0	1.0	1.0	1.0
		$SM(\overline{X})_{0.2}$	1.0	1.2	1.7	2.2	1.0	1.0	1.0	1.0	1.1
		$SM(\overline{X})_{0.6}$	1.1	1.4	2.2	3.1	1.0	1.0	1.0	1.0	1.0
		$SM(\overline{X})_1$	1.1	1.5	2.5	3.4	1.0	1.0	1.0	0.9	1.0
	10	CP	1.1	1.2	1.5	1.9	1.1	1.1	1.2	1.5	1.9
		$S\overline{\overline{X}}_{0.6}$	1.1	1.3	1.7	2.2	1.0	1.0	1.0	1.1	1.2
		$SM(\overline{X})_{0.2}$	1.0	1.1	1.4	1.8	1.0	1.0	1.0	1.1	1.2
		$SM(\overline{X})_{0.6}$	1.0	1.3	1.7	2.3	1.0	1.0	1.0	1.1	1.2
		$SM(\overline{X})_1$	1.0	1.3	1.8	2.4	1.0	1.0	1.0	1.0	1.1
Single step	5	CP	15.1	90.7	99.3	99.8	1.0	1.7	1.3	0.2	0.1
		$S\overline{\overline{X}}_{0.6}$	6.9	53.7	89.4	96.1	1.0	1.1	1.5	2.2	3.0
		$SM(\overline{X})_{0.2}$	7.9	54.7	80.0	87.1	1.0	1.2	1.5	1.6	1.6
		$SM(\overline{X})_{0.6}$	6.8	55.6	91.9	97.8	1.0	1.1	1.2	1.2	1.2
		$SM(\overline{X})_1$	3.9	30.7	78.8	95.2	1.0	1.0	1.1	1.1	1.1
	10	CP	36.1	98.7	99.9	100.0	1.0	2.1	0.3	0.0	0.0
		$S\overline{\overline{X}}_{0.6}$	15.5	83.5	98.3	99.8	1.0	1.2	2.0	3.7	5.5
		$SM(\overline{X})_{0.2}$	18.3	75.3	90.6	96.1	1.0	1.3	1.6	1.6	1.6
		$SM(\overline{X})_{0.6}$	15.5	86.2	99.3	100.0	1.0	1.1	1.2	1.2	1.2
		$SM(\overline{X})_1$	7.9	66.1	98.9	100.0	1.0	1.0	1.1	1.1	1.1
Multiple step	5	CP	4.1	28.6	48.3	53.1	1.2	1.9	7.0	13.9	16.2
		$S\overline{\overline{X}}_{0.6}$	5.6	43.9	81.9	91.4	1.0	1.1	1.8	3.3	5.0
		$SM(\overline{X})_{0.2}$	6.8	50.1	78.7	86.6	1.0	1.4	4.3	7.9	9.7
		$SM(\overline{X})_{0.6}$	5.6	47.9	88.7	96.5	1.0	1.1	1.4	1.8	2.1
		$SM(\overline{X})_1$	3.4	26.1	73.3	92.9	1.0	1.0	1.2	1.2	1.2

TABLE 3. *Continued*

Scenario	n	$\hat{\mu}$	TAP δ_I 0.4	1	1.6	2	FAP δ_I 0	0.4	1	1.6	2
	10	CP	8.8	43.8	54.5	55.7	1.1	2.7	11.5	16.1	16.4
		$S\overline{\overline{X}}_{0.6}$	12.2	74.2	94.6	97.9	1.0	1.2	2.7	6.4	10.0
		$SM(\overline{X})_{0.2}$	15.6	73.1	90.1	95.3	1.0	1.9	7.0	10.6	12.3
		$SM(\overline{X})_{0.6}$	12.5	81.3	98.6	99.8	1.0	1.2	1.6	2.5	3.1
		$SM(\overline{X})_1$	6.6	59.8	97.8	99.8	1.0	1.1	1.2	1.2	1.2

- When there are localized disturbances, $SM(\overline{X})_1$ shows the best performance because it has the highest TAP and lowest FAP values. Note that $SM(\overline{X})_{0.6}$, which is based on a robust initial estimator, detects more unacceptable observations than $S\overline{\overline{X}}_{0.6}$. This is because the robust Phase I chart limits are not biased by any contaminations.

- The proposed estimation methods detect very few diffuse disturbances. This is not surprising given that they lack an effective way of identifying outliers within samples. The methods should be augmented by an individuals chart or similar procedure. This is an issue for future research.

- When there is a single step shift, the CP method performs best, followed by the Phase I EWMA charts. The Shewhart chart performs poorly in this situation, which is to be expected as this chart is especially designed to detect individual, scattered disturbances.

- When there are multiple step shifts, the CP method, which we use, runs into trouble as it is designed to detect a single shift. A solution for this could be the use of a CP method that recursively identifies multiply changepoints. The Phase I chart based on a nonrobust estimator, $S\overline{\overline{X}}_{0.6}$, deletes too many in-control observations. The EWMA chart with $\lambda_\text{I} = 0.6$ performs best.

4. Phase II Performance

Phase I estimators are used to design Phase II control charts. In this section, we evaluate EWMA control charts in Phase II that are based on estimated parameters when the Phase I data may or may not be contaminated.

Recall that the Phase II observations, Y_{it} with $i = 1, 2, \ldots, n$ and $t = 1, 2, \ldots$, are independently and identically $N(\mu, \sigma^2)$ distributed if the process is in control. We model out-of-control Phase II data as $N(\mu + \delta_\text{II}\sigma, \sigma^2)$, where δ_II is a constant and the index 'II' indicates Phase II data.

4.1. Design of the Phase II EWMA Control Chart

The Phase II EWMA control chart consists of the EWMA statistic

$$Z_t = (1 - \lambda_\text{II})Z_{t-1} + \lambda_\text{II}\overline{Y}_t,$$

with $Z_0 = \hat{\mu}$, and control limits

$$\widehat{\text{UCL}}_t/\widehat{\text{LCL}}_t = \hat{\mu} \pm L_\text{II}\frac{\hat{\sigma}}{\sqrt{n}}\sqrt{\frac{\lambda_\text{II}}{2 - \lambda_\text{II}}[1 - (1 - \lambda_\text{II})^{2t}]},$$

where $\hat{\mu}$ and $\hat{\sigma}$ are the Phase I estimates of μ and σ. We consider seven EWMA Phase II control charts, each based on one of the location estimators ($\hat{\mu}$) presented in Section 2. For a fair comparison of the proposed location estimation methods, the standard deviation for each control chart is determined by a robust estimator described in Tatum (1997) (see Section 2.3).

Furthermore, λ_II is set equal to 0.13, which Crowder (1989) recommended as an optimal smoothing constant to detect a shift size of $\delta_\text{II} = 1$. Note that λ_II differs from the λ_I used in Phase I, as Phase I is used for exploratory data-analysis purposes. We take the values for L_II from Jones (2002): $L_\text{II} = 2.89$ for $n = 5$ and $L_\text{II} = 2.92$ for $n = 10$. These values are determined such that the EWMA control chart has an average run length of approximately 370 when the process is in control.

4.2. Performance Measures and Simulation Procedure

The Phase II performance of a control chart can be expressed in terms of the probability distribution of the run length (RL), which is a stochastic variable indicating the number of samples before Z_t falls outside the control limits, i.e., a signal that the process may be out of control. A common measure of control chart performance is the expected value of the RL, i.e., the average run length (ARL). It is desirable to have a high ARL when the process is stable and a low ARL when the process mean has shifted to $\mu + \delta_{II}\sigma$ with $\delta_{II} \neq 0$. Jones et al. (2001) gave a very insightful account of the performance of the control chart with estimated limits. In this case, a distinction is made between the conditional and unconditional run length distribution. The conditional distribution is the distribution of RL given the Phase I parameter estimates $\widehat{\mu}$ and $\widehat{\sigma}$. In order to evaluate the overall behavior of the EWMA control charts, we consider the unconditional run length distribution, which averages over the variability of the parameter estimates $\widehat{\mu}$ and $\widehat{\sigma}$.

In order to evaluate the effect on the Phase II EWMA control chart of contaminated Phase I data and the estimation methods, we estimate the average ARL of the unconditional run-length distribution by means of simulation. In order to obtain these values, we use the following simulation procedure: first, $k = 50$ samples of size $n = 5, 10$ are drawn from the same Phase I disturbance scenarios used to assess the MSE, with $\delta_I = 1$. Then $\widehat{\mu}$ and $\widehat{\sigma}$ are calculated from the data and the control limits are computed. Observations from $N(\delta_{II}, 1)$ are drawn until the associated Z_t falls outside the control limits. The corresponding run length equals t. The calculations are made for $\delta_{II} = 0, 0.1, 0.2, 0.3, 0.4$. The entire procedure is repeated for $R = 200,000$ simulation runs. The ARL is computed by averaging over all $200,000$ RLs and the results are presented in Table 4.

4.3. Phase II Results

First, consider the situation where the Phase I data are stable (first part of Table 4). We see that, for acceptable data, Phase II control chart performance is similar across all estimators (i.e., they have similar in-control and out-of-control ARLs), except for the control chart based on the robust point estimator $M(\overline{X})$, which falls short. This confirms that it is unadvisable to use a robust point estimator: if there are no special causes in Phase I, useful information is lost, resulting in a less powerful control chart. It is better to use a Phase I procedure that only trims Phase I observations that are considered unacceptable. When no data anomalies are found, the number of falsely deleted samples is limited.

When there are localized disturbances, the best Phase II performance is achieved by the screening estimators ($S\overline{\overline{X}}_{0.6}$, $SM(\overline{X})_{0.2}$, $SM(\overline{X})_{0.6}$, and $SM(\overline{X})_1$), and the control chart based on the robust point estimator $M(\overline{X})$. The EWMA control chart based on $\overline{\overline{X}}$ is less effective as this traditional estimator is influenced by outliers, which then also affect the resulting Phase II limits. The EWMA control chart based on CP does not work very well: this estimator deletes too many samples on the assumption that the disturbances will endure. Unfortunately, for $n = 5$, all EWMA control charts are ARL biased. Furthermore, none of the Phase II control charts performs well when there are diffuse disturbances because these charts are typically designed to detect sample shifts instead of individual outliers.

In the presence of a single step shift, CP performs best followed by the Phase I EWMA charts, ($S\overline{\overline{X}}_{0.6}$, $SM(\overline{X})_{0.2}$, and $SM(\overline{X})_{0.6}$). Note that the Phase I Shewhart chart ($SM(\overline{X})_1$) is outperformed by the EWMA and CP methods because the Shewhart chart has no memory and does not make use of the time sequence of the observations.

Finally, in the case of multiple step shifts, the CP method does not work well at all: the resulting control chart has a relatively low ARL for the acceptable situation and high ARL when the process is out of control, which is not desirable. This is as expected because the CP method is designed for single step shifts. In the multiple step shift scenario, the Phase I EWMA charts ($S\overline{\overline{X}}_{0.6}$, $SM(\overline{X})_{0.2}$, and $SM(\overline{X})_{0.6}$) perform best.

To summarize, the type of disturbance and estimation method used in Phase I strongly determine the performance of the Phase II EWMA control chart. We recommend the Shewhart chart in Phase I for scattered observations, the changepoint method whenever single step changes are likely, and the EWMA chart with λ_I set at around 0.6 when there are multiple step changes or when there is uncertainty about the type, the length, or the magnitude of the disturbance.

TABLE 4. Average ARL for the EWMA Control Chart in Phase II Based on Alternative Location Estimators
for $k = 50$ Samples of Size $n = 5, 10$

				Average ARL of the unconditional run-length distribution				
Phase I			In-control	Out-of-control Phase II Data				
Scenario	n	$\widehat{\mu}$	$\delta_{\mathrm{II}} = 0$	$\delta_{\mathrm{II}} = 0.1$	$\delta_{\mathrm{II}} = 0.2$	$\delta_{\mathrm{II}} = 0.3$	$\delta_{\mathrm{II}} = 0.4$
In control	5	$\overline{\overline{X}}$	374	210	61	22	12
		$M(\overline{X})$	333	214	73	25	12
		CP	370	210	63	23	12
		$S\overline{\overline{X}}_{0.6}$	370	212	62	23	12
		$SM(\overline{X})_{0.2}$	369	210	62	23	12
		$SM(\overline{X})_{0.6}$	367	211	63	23	12
		$SM(\overline{X})_1$	365	212	63	23	12
	10	$\overline{\overline{X}}$	371	129	26	11	6
		$M(\overline{X})$	333	143	30	11	6
		CP	367	132	27	11	6
		$S\overline{\overline{X}}_{0.6}$	367	131	27	11	6
		$SM(\overline{X})_{0.2}$	366	132	26	11	6
		$SM(\overline{X})_{0.6}$	362	132	26	11	6
		$SM(\overline{X})_1$	363	132	27	11	6
Localized	5	$\overline{\overline{X}}$	215	338	213	72	25
($\delta_{\mathrm{I}} = 1$)		$M(\overline{X})$	273	294	153	52	20
		CP	217	325	207	75	27
		$S\overline{\overline{X}}_{0.6}$	260	319	169	55	21
		$SM(\overline{X})_{0.2}$	252	324	177	58	21
		$SM(\overline{X})_{0.6}$	271	311	158	51	19
		$SM(\overline{X})_1$	279	308	152	48	19
	10	$\overline{\overline{X}}$	152	313	149	33	12
		$M(\overline{X})$	270	244	66	17	8
		CP	167	286	139	40	13
		$S\overline{\overline{X}}_{0.6}$	264	252	71	18	8
		$SM(\overline{X})_{0.2}$	251	263	76	19	8
		$SM(\overline{X})_{0.6}$	294	220	54	15	7
		$SM(\overline{X})_1$	308	211	49	14	7
Diffuse	5	$\overline{\overline{X}}$	287	517	296	82	26
($\delta_{\mathrm{I}} = 1$)		$M(\overline{X})$	297	458	296	97	30
		CP	288	510	297	84	28
		$S\overline{\overline{X}}_{0.6}$	291	506	296	83	27
		$SM(\overline{X})_{0.2}$	289	509	295	83	27
		$SM(\overline{X})_{0.6}$	292	507	294	84	27
		$SM(\overline{X})_1$	293	505	294	82	27

TABLE 4. *Continued*

				Average ARL of the unconditional run-length distribution				
Phase I			In-control	Out-of-control Phase II Data				
Scenario	n	$\widehat{\mu}$	$\delta_{II} = 0$	$\delta_{II} = 0.1$	$\delta_{II} = 0.2$	$\delta_{II} = 0.3$	$\delta_{II} = 0.4$
	10	$\overline{\overline{X}}$	173	511	178	31	12
		$M(\overline{X})$	198	454	196	37	12
		CP	177	504	180	32	12
		$S\overline{\overline{X}}_{0.6}$	180	504	180	32	12
		$SM(\overline{X})_{0.2}$	177	502	181	32	12
		$SM(\overline{X})_{0.6}$	178	501	180	32	12
		$SM(\overline{X})_{1}$	180	501	180	32	12
Single step $(\delta_I = 1)$	5	$\overline{\overline{X}}$	210	371	210	61	22
		$M(\overline{X})$	278	302	148	48	18
		CP	349	209	69	25	13
		$S\overline{\overline{X}}_{0.6}$	312	298	119	37	16
		$SM(\overline{X})_{0.2}$	306	302	125	39	16
		$SM(\overline{X})_{0.6}$	317	284	112	35	15
		$SM(\overline{X})_{1}$	282	321	145	44	18
	10	$\overline{\overline{X}}$	130	370	130	26	11
		$M(\overline{X})$	276	249	60	16	8
		CP	361	132	26	11	6
		$S\overline{\overline{X}}_{0.6}$	326	200	39	13	7
		$SM(\overline{X})_{0.2}$	316	215	44	14	7
		$SM(\overline{X})_{0.6}$	338	175	35	12	7
		$SM(\overline{X})_{1}$	313	211	44	14	7
Multiple steps $(\delta_I = 1)$	5	$\overline{\overline{X}}$	220	266	193	97	39
		$M(\overline{X})$	257	264	156	67	28
		CP	247	230	143	77	43
		$S\overline{\overline{X}}_{0.6}$	287	264	137	57	24
		$SM(\overline{X})_{0.2}$	295	262	129	52	22
		$SM(\overline{X})_{0.6}$	297	257	127	51	22
		$SM(\overline{X})_{1}$	268	269	154	66	27
	10	$\overline{\overline{X}}$	180	222	145	62	21
		$M(\overline{X})$	254	222	82	25	10
		CP	242	159	80	48	32
		$S\overline{\overline{X}}_{0.6}$	299	197	60	21	9
		$SM(\overline{X})_{0.2}$	305	196	56	18	9
		$SM(\overline{X})_{0.6}$	324	175	44	15	7
		$SM(\overline{X})_{1}$	293	201	62	20	9

5. Conclusion

We have considered several Phase I estimation methods for situations where scattered as well as sustained shifts might be present. We have studied the effectiveness of a Phase I exploratory data analysis stage in terms of the accuracy of the resulting estimates and the proportion of successfully identified unacceptable samples. Moreover, we have investigated the impact of data contaminations and the estimators used in Phase I on the performance of the Phase II EWMA control chart.

Our study shows that data anomalies can have a huge impact on the quality of the Phase I analysis as well as on the power of the resulting Phase II EWMA control chart. There is considerable difference in the performance of the Phase I estimation methods. We have the following recommendations:

First, we recommend the use of a changepoint or SPC-based method instead of a single point estimator to arrive at a parameter estimate for the Phase II limits. Such methods make it possible to perform a Phase I data analysis and 'learn from the data' before any monitoring takes place. A learning stage will improve the quality of the Phase II monitoring.

Second, we have seen that most methods work well in one specific situation: when there might be scattered contaminations, the Shewhart chart works best but, when there are sustained shifts, the use of an EWMA chart or changepoint method is more appropriate. These methods take into account the time sequence in the samples. When multiple step changes are likely or the type of disturbance is unknown, we recommend the Phase I EWMA chart. The smoothing constant of the EWMA chart, λ_I, should ideally be set at around 0.6. A low value overemphasizes the detection of small shifts while a high value approaches the performance of the Phase I Shewhart chart.

The methods based on EWMA charts not only work well in the multiple step scenario but also in the other contamination scenarios. These estimators provide near-best estimates of the location in the presence of any pattern of Phase I contaminations.

Finally, we recommend the use of a two-step procedure, namely a robust estimator to estimate the location and construct the Phase I control chart, and an efficient estimator for post-screening estimation. The use of a robust estimator for the Phase I chart ensures that Phase I limits are not too sensitive to any disturbances, limiting the incorrect deletion of clean data, while the use of an efficient estimator subsequent to screening ensures that final estimates are efficient under stability as well.

Acknowledgments

Appendix

TABLE 5. $E(\mathrm{LRT}(\tau))$, the Normalizing Constants for LRT

| τ | n | | τ | n | | τ | n | | τ | n | |
	5	10		5	10		5	10		5	10
1			14	2.03	2.02	27	2.02	2.02	40	2.04	2.02
2	2.21	2.11	15	2.03	2.02	28	2.03	2.02	41	2.04	2.03
3	2.14	2.06	16	2.03	2.02	29	2.03	2.02	42	2.05	2.03
4	2.10	2.05	17	2.03	2.02	30	2.03	2.02	43	2.06	2.04
5	2.08	2.04	18	2.03	2.02	31	2.03	2.02	44	2.07	2.04
6	2.07	2.04	19	2.03	2.02	32	2.03	2.02	45	2.08	2.04
7	2.06	2.04	20	2.03	2.02	33	2.03	2.02	46	2.10	2.05
8	2.05	2.03	21	2.03	2.02	34	2.03	2.02	47	2.14	2.06
9	2.04	2.03	22	2.02	2.02	35	2.03	2.02	48	2.21	2.10
10	2.04	2.02	23	2.02	2.02	36	2.03	2.02	49		
11	2.03	2.02	24	2.02	2.02	37	2.03	2.02	50		
12	2.03	2.02	25	2.02	2.02	38	2.03	2.02			
13	2.03	2.02	26	2.02	2.02	39	2.03	2.02			

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