QUALITY AND RELIABILITY ENGINEERING INTERNATIONAL
Qual. Reliab. Engng. Int. 2006; **22**:239–249
Published online 16 December 2005 in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/qre.700

Research

Individuals Charts and Additional Tests for Changes in Spread

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A number of authors have indicated that it is not a good idea to use the moving range chart (MR-chart) with an individuals chart to detect shifts in the spread. However, the combination of these charts is still used in practice and even presented as 'best practice' in some cases. In addition, some more recent articles present arguments that justify the use of the additional MR-chart in specific situations. In this paper we investigate these arguments and add to the literature by providing two more reasons not to use the MR-chart. First, the merit of using an MR-chart has never been evaluated before in the context of runs rules. Earlier papers investigated the change in performance as a result of adding a MR-chart to a bare individuals chart. We show that relative to existing well-known runs rules, there is no advantage in using a close alternative to the MR-chart. Secondly, we investigate the suggestion of some of the proponents of the MR-chart that its weak performance is due to a bad design. We show that this is not the case. We evaluate the average run length performance of the combination of an individuals chart and a MR-chart under the most favorable circumstances for several out-of-control situations by optimizing the design of the two charts for each situation. Our results show that even this 'best-case' performance of the combination is hardly better than that of the individuals chart alone.

KEY WORDS: individuals chart; *MR*-chart; runs rules

1. INTRODUCTION

Control charts for individual observations are frequently used in industry. Such charts are useful because in some applications it may be impossible to collect more than one observation per sample. For example, this is often the case in process industries, where parameters such as temperature and concentration are monitored. In such cases the $\bar{X}$–R chart (or affiliates) cannot be used as it is impossible to calculate the within-sample variation when the sample size equals one.

A disadvantage of the individuals chart is that every departure from the in-control situation is signalled on only one chart, whereas the $\bar{X}$–R chart monitors changes in the process mean and the process variation separately.

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Received 3 December 2003
Revised 10 November 2004

For this reason, the moving range chart (*MR*-chart) has been suggested to accompany the individuals chart for detecting changes in process variation. In a later section we cite several authors that have investigated this suggestion and found that there are essentially no advantages to using the *MR*-chart with the *X*-chart, as this does not sufficiently improve monitoring performance. In spite of these findings some recent publications recommend the *MR*-chart again:

- Adke and Hong[1] recommend a simple technique as 'an equivalent substitute for the *MR*-chart without the disadvantage of the *MR*-chart';
- Albin *et al.*[2] advise to 'add the 2/3 opposite side run rule' (which is a modification of the *MR*-chart) 'if it is critical to detect smaller shifts in the standard deviation';
- Amin and Ethridge[3] 'recommend the combined *X*–*MR* procedure to those practitioners who choose to use Shewhart-type control charts for individual measurements'.

In addition, some tutorial articles also recommend the *X*–*MR* chart. For example, Ward[4] discusses a problem encountered by Bob, a quality control manager for a company that makes glass containers, and recommends solving Bob's problem with an *X*–*MR* chart in Ward[5]. Another example is Conklin[6], who recommends the *X*–*MR* chart to newcomers to Six Sigma.

The present paper is motivated by a discussion that arose at Philips Semiconductors in Stadskanaal, a QS9000 certified manufacturer of medium power diodes, where both authors have been working. Before the QS9000 audit Philips was mainly using individuals charts, as advised by Roes and Does[7]. During the QS9000 audit, the auditor urgently advised to 'improve' the power of the statistical process control (SPC) system by introducing supplementary Western Electric runs rules. A set of runs rules was selected to oblige the auditor, and as a result the charts gave many additional signals. However, it remained unclear whether these extra signals were caused by either an increased probability of a 'false' signal, or by out-of-control situations that were not detected before (such as changes in the spread). Moreover, the question was raised whether it was useful to add a *MR*-chart to the existing system for faster detection of changes in the spread.

This article can be divided into two parts. In the first part we investigate whether the *MR*-chart would be useful in the particular runs-rules context of Philips Semiconductors in Stadskanaal. To this end we evaluate the use of an additional runs rule introduced by Page[8] that can be considered as a slight modification of the standard *MR*-chart. Using Page's runs rule, monitoring for a change in the spread fits smoothly within the runs-rules framework, and an additional chart for monitoring the spread is not needed.

We investigate the change in performance when Page's runs rule is added to the particular set of runs rules that were used at Philips Semiconductors in Stadskanaal. The results indicate that only for specific out-of-control situations—a small increase of the spread—is some minor improvement attained. To the best of our knowledge the performance of an additional *MR*-chart has never been evaluated before in the context of runs rules.

In the second part of this article, we turn from the specific setting of Philips Semiconductors to the more general question of whether it is useful to add a *MR*-chart to a individuals chart. We summarize the discussion in recent literature on this point of controversy. Subsequently, we focus on the claim of some of the proponents of the combined *X*–*MR* procedure, and demonstrate that its weak performance is not due to a bad design.

Throughout this article, we compare the performance of the different monitoring schemes assuming that they are applied to independently and identically distributed normal random variables. We computed the average run length (ARL) of an individuals chart that is combined with Page's runs rule using the method proposed by Page[8]. For numerical evaluation of the ARL of the combined *X*–*MR* chart, we developed a more flexible version of the integral equation approach of Crowder[9,10]. The technical details can be found in Trip and Wieringa[11].

2. RULES FOR DETECTION OF OUT-OF-CONTROL SITUATIONS

The standard rule of an individuals chart to issue a signal is as follows.

Rule 1. A control limit is exceeded.

Usually, the so-called 3σ-limits are taken as the control limits. The following three of the many Western Electric runs rules were introduced at Philips Semiconductors Stadskanaal to improve the power of the control chart.

Rule 2. A signal is issued when two out of three measurements are in the same warning zone (the region between a warning limit—usually taken as the so-called 2σ-limits—and the corresponding control limit).

Rule 3. A run of six consecutive measurements either increasing or decreasing.

Rule 4. A run of nine measurements above or below the central line (CL).

The reason for choosing exactly these three runs rules is rather arbitrary. In fact they were selected because it was felt that they were easy to comprehend by operators. This is an important argument because operators may have to stop the process when a signal is given: it certainly helps when they are convinced that there is really something happening to the process.

An individuals chart is sometimes supplemented with a *MR*-chart, plotting the successive ranges of two consecutive measurements, issuing a signal when the upper control limit is exceeded. The *MR*-chart operates in a similar manner to the following runs rule.

Rule A. Two successive measurements in the opposite warning zones.

Page[8] discussed Rule A (and related rules) long before the development of the *MR*-chart:

> 'the occurrence [. . .] of two near samples outside opposite warning lines points to an increase in the spread of the distribution.'

From the results in the second part of this paper it can be inferred that Rule A is less sensitive to out-of-control situations than the *MR*-chart, but the differences in behavior are small.

Another of Page's rules was studied by Albin *et al.*[2]. They investigated the performance of a direct alternative to Rule 2: two out of three measurements in opposite warning zones. However, Rule A is a more direct translation of the *MR*-chart and is simple to use: there is no need for an additional chart or for a movable transparency, as suggested by Adke and Hong[1]. A signal due to Rule A is comparable to exceeding the upper control limit of a *MR*-chart. A lower control limit can be computed for the *MR*-chart as well. However, this limit is not useful; neither in theory (Wieringa[12] showed that additional power in the one-sided case is larger than in the two-sided case) nor in practice. The lower control limit usually is so small that in practice a signal is issued only when two consecutive measurements are equal, which happens much more often in real life than in theory due to rounding off of the data.

The purpose of the following section is to evaluate the change in performance of the individuals chart when Rule A is added to the existing set of runs rules. The basic question is whether Rule A helps to identify out-of-control processes without generating too many false alarms.

3. PERFORMANCE OF THE RUNS RULE FOR VARIATION

In this section, we study the performance of the individuals chart in combination with the aforementioned runs rules through Monte Carlo simulations. The method of Champ and Woodall[13] for calculating exact run-length probabilities cannot be used in this case because Rule 3 does not fit into the class of runs rules they considered. We computed the control and warning limits assuming an in-control mean of μ_0, and an in-control standard deviation of σ_0. We simulated data from normal distributions, for several combinations of shifts in the mean and shifts in the standard deviation. The size of the shifts in the mean ranges from zero up to 2.5 units of σ_0. The simulated standard deviations range from $0.5\sigma_0$ up to $3\sigma_0$. We simulated five series of 1 000 000 consecutive measurements for each situation. This number of iterations proved to be large enough for the standard errors to

Table I. ARLs without and with Rule A

	Without Rule A $(\mu-\mu_0)/\sigma_0$						With Rule A $(\mu-\mu_0)/\sigma_0$					
σ/σ_0	0	0.5	1.0	1.5	2.0	2.5	0	0.5	1.0	1.5	2.0	2.5
0.50	433	23.5	10.1	8.08	4.24	2.17	433	23.5	10.1	8.08	4.24	2.17
0.75	417	47.2	13.3	7.10	3.88	2.28	411	47.1	13.3	7.10	3.88	2.28
1.00	151	43.6	13.0	6.19	3.55	2.29	135	42.5	13.0	6.18	3.55	2.29
1.25	38.7	22.3	9.93	5.31	3.30	2.29	34.2	21.0	9.78	5.29	3.30	2.29
1.50	16.0	12.3	7.39	4.59	3.11	2.28	14.5	11.5	7.18	4.55	3.10	2.28
2.00	6.25	5.73	4.65	3.59	2.79	2.23	5.86	5.42	4.48	3.51	2.76	2.22
2.50	3.89	3.75	3.38	2.94	2.51	2.15	3.72	3.60	3.27	2.87	2.47	2.13
3.00	2.93	2.87	2.72	2.50	2.27	2.04	2.83	2.78	2.64	2.45	2.23	2.02

Table II. ARL results of Albin *et al.*[2]

Individuals chart with . . .	ARL			Compared with 'in-control'	
	In-control	$\sigma/\sigma_0 = 1.2$	$\sigma/\sigma_0 = 1.5$	$\sigma/\sigma_0 = 1.2$	$\sigma/\sigma_0 = 1.5$
No rules rules	370	79	22	21.4%	5.9%
2/3 same side	208	52	16	25.0%	7.7%
2/3 opposite sides	207	52	16	25.1%	7.7%

be small (typically less than 0.5% of the ARL value) for all interesting situations‡. The resulting ARLs of these five series are presented in Table I. We adopted the convention that all rules are reset after a signal has been given—which agrees with the sound practice of bringing the process in-control before it is continued.

The in-control ($(\mu-\mu_0)/\sigma_0 = 0,\ \sigma/\sigma_0 = 1$) ARL decreases from 151 to 135 (10.6%) when Rule A is added to the SPC system with runs Rules 1–4. The only situation with a larger percentage decrease in ARL is the out-of-control situation where there has been a slight increase in the process variation ($(\mu-\mu_0)/\sigma_0 = 0,\ \sigma/\sigma_0 = 1.25$). In this case the ARL decreases from 38.7 to 34.2 (11.6%). For all other simulated combinations of shifts in μ and σ the decrease in ARL is less than 10%. Hence, the improvement due to Rule A is maximal when the process variation is slightly larger than in the in-control situation. However, the gain is only marginal.

Albin *et al.*[2] performed a similar analysis, but their conclusion differs from ours. As mentioned in the previous section, they investigated the virtue of adding a '2/3 opposite' rule (i.e. a signal is issued when two out of three measurements are in opposite warning zones) to an individuals chart. Their ARL results are reprinted in the middle columns of Table II.

Based on the results of Table II they recommend that the 2/3 opposite side rule should be added for detection of smaller shifts in the standard deviation. In our opinion this conclusion is not correct. Columns 5 and 6 of Table II show the out-of-control ARLs as percentages of the corresponding in-control ARLs. From these percentages we conclude that the addition of any of the runs rules is *harmful*, as the largest relative ARL reduction for out-of-control situations is achieved without runs rules.

Next, we investigate another potential contribution of Rule A, namely that it can act as a simple diagnostic check to detect shifts in the process variance, as opposed to shifts in the location or trends that may be signalled by other rules. In order to investigate this, we compute the share of signals that are due to Rule A under various out-of-control situations. To this end, we simulate several combinations of shifts in the mean and in the process variation, and record the share of the four runs rules in the total number of out-of-control signals. For evaluation of the effects of the individual runs rules, we adopt the convention that each signal is contributed to one rule only: the rule with the shortest window. Hence, if two rules signal at the same time, Rule 1 always gets first priority, then Rule A, and so on.

‡The maximum of 1.6% is attained in the upper left-hand corner of the table.

Table III. Percentage of signals from Rule A

	$(\mu - \mu_0)/\sigma_0$					
σ/σ_0	0	0.5	1.0	1.5	2.0	2.5
0.50	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
0.75	1.3%	0.1%	0.0%	0.0%	0.0%	0.0%
1.00	11.4%	2.8%	0.4%	0.1%	0.0%	0.0%
1.25	*13.1%*	6.7%	1.9%	0.4%	0.1%	0.0%
1.50	11.4%	8.0%	3.5%	1.3%	0.4%	0.1%
2.00	7.9%	6.8%	4.7%	2.7%	1.4%	0.7%
2.50	5.6%	5.2%	4.2%	3.0%	2.0%	1.2%
3.00	4.0%	3.9%	3.4%	2.7%	2.1%	1.5%

Table III shows that Rule A is responsible for 11.4% of the signals when the process is in control. However, for nearly all out-of-control states the share of signals from Rule A is less than 11.4%. This is a rather devastating conclusion for Rule A, because it means that the signaling share is higher in in-control situations than in nearly all out-of-control situations. The exception is when the process has up to 50% higher spread and unchanged location. Unless at least half of the out-of-control situations belong to this state, the use of Rule A is harmful to the overall performance of the system.

Trip[14] investigated the practical problem of how operators should interpret the signals from the different runs rules. He simulated several combinations of shifts in the mean and in the standard deviation, and looked at the number of signals from the different runs rules. Based on this knowledge, he was able to assign the most likely out-of-control situation to a signal from a specific runs rule. He concluded that it is nearly impossible to designate a specific out-of-control situation for which Rule A is the most suitable runs rule. The best conclusion from a signal from Rule A will be that the process mean is still on target; there might be a small increase in variation. This knowledge provides useful information for the design of out-of-control action plans (OCAPs) (see Sandorf and Bassett[15]). At Philips in Stadskanaal, Rule A was not seen as beneficial in this respect, and is therefore not implemented.

4. DISCUSSION OF RECENT LITERATURE

In the previous sections, the additional *MR*-chart was discussed from a practical point of view. Instead of the *MR*-chart itself, we examined an alternative runs rule, that behaves similarly (which can be inferred from the results in the next section). In the remaining part of this paper, we perform a theoretical analysis of the value of adding a *MR*-chart, and confirm our earlier conclusions. Before doing so, we start with a discussion of recent literature because there is controversy on the usefulness of an additional *MR*-chart.

In this discussion we do not focus on handbooks or textbooks such as Duncan[16], Ryan[17], Wheeler and Chambers[18], Wetherill and Brown[19] and Montgomery[20]. These usually do not provide a clear recommendation whether or not to use the *MR*-chart. Ryan[17](p. 160), for example, acknowledges that 'the moving range chart has considerable shortcomings' but notes that 'the moving ranges might still be compared against the control limits'. Also Montgomery[20] advises to rely primarily on the individuals chart as the moving range chart cannot provide useful information, but believes that 'as long as the analyst is careful in interpretation [. . .] little trouble will ensue from plotting both charts'.

Also, we do not focus on alternative individuals charts that are developed for monitoring the spread of individual observations such as CUSUM charts (see, e.g., Hawkins[21]), EWMA-charts (see, e.g., Wortham and Ringer[22] and Acosta-Mejía and Pignatiello[23] for an ARL comparison of such charts) or the *LR*-chart (Braun[24]). Instead, we focus on some articles that provide a contribution to the discussion on the usefulness of the *MR*-chart.

An outspoken opponent of the X–MR chart is Nelson[25,26]. He strongly advises against the use of the MR-chart based on two arguments. Firstly, he argues that interpretation is complicated by the serial correlation of successive points on the MR-chart. His second argument is that the individuals chart itself already contains all the information available.

Roes *et al.*[27] are also opposed to the use of the MR-chart. They computed the conditional probability (assuming independence of the observations) of observing a signal on the MR-chart, given that the X-chart itself does not signal. These probabilities are small for the out-of-control situations. Therefore, they concluded that the contribution of the MR-chart to the power of discovering an out-of-control situation is small. Wieringa[12] agrees with this conclusion, but not with the argument. A small probability of a signal on the MR-chart may be due to a poor design of the chart, e.g. the limits are too wide. He argues that the *differences* between the probability of a signal in the in-control situation and the probabilities of a signal in various out-of-control situations are important. A control chart is useful if these differences are large. For the additional MR-chart, these differences are small.

Rigdon *et al.*[28] followed Wetherill and Brown[19] and examined ranges of not just two, but also of three and four consecutive observations. They selected control limits for the combined X–MR chart so that the in-control ARL was the same as for the X-chart alone. Their conclusion was that for shifts in the process mean the X-chart alone is more effective, while this chart is about as equally effective in detecting changes in the process variability as the combined X–MR chart.

Surprisingly, the three most recent contributions to this discussion are more positive regarding the usefulness of the MR-chart. They present arguments as to why it is useful to consider the MR-chart in some specific situations.

Adke and Hong[1] derived a lower bound for the probability that a shift is detected on the MR-chart during an interval with its length equal to the ARL of the individuals chart, conditional on its non-detection on the X-chart. They found that this lower bound equals 0.1152 for a shift in the standard deviation of 1–1.5. They concluded that ‘the successive differences used on the MR-chart do contain useful information on shifts in the process variance’ as 0.1152 ‘obviously cannot be dismissed as small’. What they fail to note is that the lower bound of this probability is even larger in the in-control situation (namely 0.1350, see Table 2 in Adke and Hong[1]), which leads to the awkward conclusion that the MR-chart is more likely to signal during a typical in-control run of observations than in a typical out-of-control run of observations.

Albin *et al.*[2] investigated the individuals X-chart in combination with runs rules, the MR-chart, amongst others. Based on ARL values they recommend not using complementary runs rules. Small increases in the process variation are not well detected, however. If it is critical to detect such shifts, they propose the use of one of the alternative runs rules that we discussed previously. In the previous section, we argued that we do not agree with their recommendation.

Amin and Ethridge[3] believe that the relatively poor performance of the combined X–MR chart as reported by Rigdon *et al.*[28] is due to the choice of the chart parameters. Based on ARL values, they concluded that there is no disadvantage in using the X–MR procedure. We investigate this claim in the next section.

However, apart from ARL considerations they have additional reasons to recommend the use of the combined X–MR chart. They assert that

> ‘if only the X-chart is used, the user will not be able to directly distinguish between a shift in the process mean and changes in the process variability.’

However, a shift in the process mean is likely to show up in the data as a change in the level of the measurements, and also increases the probability of observing a large successive difference at the time of the shift. Thus, both charts are likely to signal an increase in the process mean. Following the same line of reasoning, we argue that an increase in the process variance is likely to lead to signals in both the X- and the MR-chart. Hence, in our opinion, both types of shifts are likely to manifest themselves in both charts, so that there is also no diagnostic value in using the combined procedure. This is confirmed by the results in the previous section, where we showed that a signal due to Rule A does not point clearly to an increase in the process variability.

Table IV. ARL values of the individuals chart

	$(\mu - \mu_0)/\sigma_0$									
σ/σ_0	0.00	0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00	4.00
1.00	370.40	281.15	155.22	81.22	43.89	14.97	6.30	3.24	2.00	1.19
1.25	60.99	53.87	39.52	26.82	18.02	8.68	4.72	2.90	2.00	1.27
1.50	21.98	20.62	17.36	13.70	10.52	6.25	3.95	2.71	2.00	1.34
2.00	7.48	7.32	6.86	6.22	5.51	4.19	3.18	2.47	1.99	1.45
2.50	4.35	4.30	4.18	3.99	3.75	3.22	2.72	2.30	1.97	1.52
3.00	3.15	3.13	3.09	3.01	2.91	2.66	2.40	2.14	1.91	1.56
4.00	2.21	2.20	2.19	2.17	2.14	2.07	1.97	1.87	1.76	1.57

Table V. ARL values of the individuals chart, combined with the MR-chart

	$(\mu - \mu_0)/\sigma_0$									
σ/σ_0	0.00	0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00	4.00
1.00	278.23	226.84	139.33	77.59	43.13	14.93	6.30	3.24	2.00	1.19
	(75.1%)	(80.7%)	(89.8%)	(95.5%)	(98.2%)	(99.8%)	(100.0%)	(100.0%)	(100.0%)	(100.0%)
1.25	47.66	43.49	34.15	24.62	17.21	8.58	4.71	2.90	2.00	1.27
	(78.1%)	(80.7%)	(86.4%)	(91.8%)	(95.5%)	(98.9%)	(99.8%)	(100.0%)	(100.0%)	(100.0%)
1.50	18.00	17.16	15.00	12.35	9.83	6.10	3.92	2.70	2.00	1.34
	(81.9%)	(83.2%)	(86.4%)	(90.2%)	(93.5%)	(97.6%)	(99.2%)	(99.8%)	(99.9%)	(100.0%)
2.00	6.59	6.48	6.15	5.67	5.11	4.01	3.11	2.45	1.99	1.44
	(88.1%)	(88.5%)	(89.6%)	(91.1%)	(92.7%)	(95.8%)	(97.8%)	(99.0%)	(99.6%)	(99.9%)
2.50	4.00	3.96	3.87	3.71	3.52	3.08	2.64	2.26	1.95	1.52
	(92.0%)	(92.1%)	(92.5%)	(93.2%)	(93.9%)	(95.6%)	(97.1%)	(98.3%)	(99.0%)	(99.7%)
3.00	2.97	2.96	2.92	2.86	2.77	2.56	2.33	2.10	1.89	1.55
	(94.4%)	(94.4%)	(94.6%)	(94.9%)	(95.3%)	(96.2%)	(97.1%)	(98.0%)	(98.6%)	(99.5%)
4.00	2.14	2.13	2.12	2.10	2.08	2.01	1.93	1.84	1.74	1.55
	(96.9%)	(96.9%)	(96.9%)	(97.0%)	(97.1%)	(97.4%)	(97.8%)	(98.2%)	(98.5%)	(99.1%)

5. THE ADDED VALUE OF AN ADDITIONAL CHART FOR THE SPREAD

In this section we investigate whether adding either Rule A or the *MR*-chart increases the power of an individuals chart for detecting out-of-control situations. Since we are mainly interested in the *MR*-chart and Rule A in combination with the individuals chart, Rules 2, 3 and 4 are not considered. This has the additional advantage that the ARL computations are simplified so that we do not have to rely on simulation results; exact ARL values are derived instead. Technical details concerning these computations can be found in Trip and Wieringa[11].

Table IV presents the ARL values of the individuals chart only. As before, a normal distribution is assumed. For the in-control situation, the mean is assumed to be μ_0, and the standard deviation is assumed to be σ_0. ARL values are computed for several out-of-control situations. The entries in Table IV are the baseline for determining the usefulness of adding either Rule A or the *MR*-chart.

Table V contains the ARL values of the individuals chart, combined with a standard one-sided *MR*-chart. Based on the derivation of the 3σ control limits for the *MR*-chart in Appendix C of Roes *et al.*[27], we set UCL_{MR}, the upper control limit of the *MR*-chart, equal to $4.65\sigma_0$. The bracketed percentages in the table express the ARL values as percentages of the corresponding values in Table IV.

Table V shows that all ARL values are smaller if the individuals chart is supplemented with the *MR*-chart. From the percentages, we conclude that the decrease in the in-control ARL is larger than the decrease in ARL for all of the out-of-control situations considered. This leads to the conclusion that the improvement in power due to the addition of the *MR*-chart is not sufficient to compensate for the increased probability of a type I error.

Table VI. 'Best-case' ARL values of the individuals-chart, combined with the *MR*-chart

	$(\mu-\mu_0)/\sigma_0$									
σ/σ_0	0.00	0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00	4.00
1.00		281.12 (100.0%) [6.0; 3.000]	155.21 (100.0%) [6.0; 3.000]	81.21 (100.0%) [6.0; 3.000]	43.89 (100.0%) [6.0; 3.000]	14.97 (100.0%) [6.0; 3.000]	6.30 (100.0%) [6.0; 3.000]	3.24 (100.0%) [6.0; 3.000]	2.00 (100.0%) [6.0; 3.000]	1.19 (100.0%) [6.0; 3.000]
1.25	57.28 (93.9%) [4.5; 3.127]	52.06 (96.6%) [4.7; 3.057]	39.30 (99.4%) [5.1; 3.011]	26.82 (100.0%) [5.6; 3.001]	18.02 (100.0%) [6.0; 3.000]	8.68 (100.0%) [6.0; 3.000]	4.72 (100.0%) [6.0; 3.000]	2.90 (100.0%) [6.0; 3.000]	2.00 (100.0%) [6.0; 3.000]	1.27 (100.0%) [6.0; 3.000]
1.50	20.53 (93.4%) [4.6; 3.084]	19.52 (94.7%) [4.6; 3.084]	16.88 (97.2%) [4.8; 3.039]	13.56 (99.0%) [5.0; 3.017]	10.50 (99.8%) [5.3; 3.005]	6.25 (100.0%) [5.8; 3.000]	3.95 (100.0%) [6.0; 3.000]	2.71 (100.0%) [6.0; 3.000]	2.00 (100.0%) [6.0; 3.000]	1.34 (100.0%) [6.0; 3.000]
2.00	7.13 (95.3%) [4.6; 3.084]	7.00 (95.6%) [4.6; 3.084]	6.62 (96.4%) [4.7; 3.057]	6.07 (97.5%) [4.8; 3.039]	5.43 (98.4%) [4.9; 3.026]	4.17 (99.6%) [5.1; 3.011]	3.17 (99.9%) [5.4; 3.003]	2.47 (100.0%) [5.7; 3.000]	1.99 (100.0%) [6.0; 3.000]	1.45 (100.0%) [6.0; 3.000]
2.50	4.21 (97.0%) [4.7; 3.057]	4.18 (97.1%) [4.7; 3.057]	4.07 (97.4%) [4.7; 3.057]	3.90 (97.8%) [4.8; 3.039]	3.68 (98.2%) [4.8; 3.039]	3.19 (99.1%) [4.9; 3.026]	2.71 (99.6%) [5.1; 3.011]	2.30 (99.9%) [5.2; 3.007]	1.97 (100.0%) [5.4; 3.003]	1.52 (100.0%) [5.9; 3.000]
3.00	3.09 (98.1%) [4.8; 3.039]	3.08 (98.1%) [4.8; 3.039]	3.03 (98.2%) [4.8; 3.039]	2.96 (98.4%) [4.8; 3.039]	2.87 (98.6%) [4.8; 3.039]	2.64 (99.0%) [4.9; 3.026]	2.38 (99.4%) [5.0; 3.017]	2.13 (99.7%) [5.1; 3.011]	1.91 (99.9%) [5.2; 3.007]	1.56 (100.0%) [5.5; 3.002]
4.00	2.19 (99.1%) [4.9; 3.026]	2.18 (99.1%) [4.9; 3.026]	2.17 (99.1%) [4.9; 3.026]	2.15 (99.2%) [4.9; 3.026]	2.12 (99.2%) [4.9; 3.026]	2.05 (99.3%) [4.9; 3.026]	1.96 (99.5%) [4.9; 3.026]	1.86 (99.6%) [5.0; 3.017]	1.76 (99.7%) [5.1; 3.011]	1.56 (99.9%) [5.2; 3.007]

When the individuals chart is supplemented with Rule A, we obtain similar results. However, the X–MR chart is a little more sensitive for detecting small increases in the spread, but the differences are small.

We conclude that adding either a standard *MR*-chart or Rule A does not improve the power of the individuals chart for detecting a shift in the spread.

As we discussed in the previous section, Amin and Ethridge[3] believe that this poor performance of the X–MR procedure relative to using the X-chart alone is due to the choice of chart parameters for the X–MR procedure. In the remainder of this section, we extend their analysis and argue that we cannot validate their claim.

For the design of the combined procedures, there are two parameters to be determined: one that fixes the control limits for the individuals chart, and one that fixes the limit(s) for the chart for the spread. There is only one restriction: the in-control ARL of the combined procedure should equal 370.4, the in-control ARL of Table IV. Hence, there are many combinations of parameters that lead to the same in-control ARL.

This makes it possible to find an 'optimal' combination of parameters for every out-of-control situation: the one that minimizes the ARL for that specific shift in the mean and standard deviation. We note that this ARL has little to no practical value, as in most practical cases it is unknown which out-of-control situation is most likely to occur. However, optimizing a combined procedure for a specific out-of-control situation provides insight into the 'best-case' performance for that situation. If we allow the control chart parameters to differ for every specific out-of-control situation, then the same procedure with fixed chart parameters cannot produce better ARL values.

A 'best-case' evaluation of the combined control chart procedure therefore proceeds as follows. The optimal combination of chart parameters is determined for various combinations of shifts in the mean and shifts in the standard deviation. In Table VI we present the 'best-case' ARL values of an individuals chart that is combined with a one-sided *MR*-chart.

The values in this table were obtained as follows. Assume that the upper limit of the *MR*-chart is set to $UCL_{MR} = R\sigma_0$ and the control limits of the individuals chart are set to $\mu_0 \pm M\sigma_0$. For the *MR*-chart parameter R ranging from 4.2, 4.3, ..., 6 the corresponding individuals chart parameter M is determined, so that the combined procedure has an in-control ARL of 370.4. For each of the resulting 19 (R, M) combinations, an ARL table like Table V is computed. The minimum ARL value of the 19 tables is selected to enter Table VI. The bracketed numbers below express the ARL values as a percentage of the corresponding ARL value of the individuals chart. The selected values of R and M are also given.

The reason for considering this range for the control limit of the *MR*-chart is as follows. For $R < 4.2$, the *MR*-chart is so sensitive that it is not possible to attain an in-control ARL-value of 370.4, no matter how large M is. For $R \geq 6$, the individuals chart is more sensitive than the *MR*-chart.

We performed an analogous analysis for Rule A and concluded that the ARL values are very similar to the corresponding values in Table VI, indicating once more that both procedures operate in a similar manner. The ARL comparisons show that the *MR*-chart performs slightly better than Rule A. However, it requires an additional chart (or a transparency-procedure, see Adke and Hong[1]), whereas Rule A can be read directly from the individuals chart. If, for whatever reason, one were to use the combined X–MR procedure for faster detection of small increases in the spread, an overall good choice for the limits is to choose the upper limit of the *MR*-chart in the range 4.5–4.7, and to choose the limits of the individuals chart so that the in-control ARL of the combined procedure equals 370.4. The consequence of this is, however, that the ARL values for pure shifts in the mean increase considerably. For example, if $(R, M) = (4.5, 3.127)$ is used, the ARL for detecting a shift in the mean of $1\sigma_0$ increases by 32.6%. Whereas, if $(R, M) = (4.6, 3.084)$ is used, this ARL increases by 20.6%. For $(R, M) = (4.7, 3.057)$, this ARL value is increased by 13.5% (note that these results cannot be inferred from Table VI).

The main conclusion that we draw from Table VI is that even 'best-case' evaluation of the combined procedures only shows slight improvement in the sensitivity for detecting shifts in the spread for a very limited number of out-of-control situations. The increase in sensitivity is largest when the standard deviation of the process changes from σ_0 to $1.5\sigma_0$: in that case the 'best-case' ARL of the X–MR chart is about 6% smaller than the ARL of the X-chart. For larger shifts in the spread the improvement due to adding an *MR*-chart is even less.

6. CONCLUSIONS

In the first part of this paper, we considered a practical situation that motivated this study. We investigated the usefulness of an additional test for the spread in an existing set of runs rules that are used to monitor individual observations. Our conclusion is that this test should not be added, as it provides no additional sensitivity for detecting out-of-control situations.

In the second part of this paper we reviewed the ongoing debate on the usefulness of an additional chart for discovering changes in the spread of individual observations and focused on the three most recent contributions. All three papers put forward arguments why the additional *MR*-chart is useful in some situations. We argue that some of these arguments are based on incorrect conclusions, and investigated the merit of two main arguments in greater detail. The first is that the relatively poor ARL performance of the X–MR chart may be due to a bad design, i.e. that the performance of the combined procedure may be improved by a better choice of chart parameters. The second argument is that, irrespective of ARL arguments, there is some diagnostic value in the additional chart for the spread that might help in finding the special cause that is responsible for an out-of-control signal.

We investigate 'best-case' performance of the individuals chart with an additional test for the spread and show that its relatively poor performance is not due to a bad choice of the chart parameters.

We also argue that adding the *MR*-chart or Rule A does not provide any help in diagnosing out-of-control situations. By examining the share of signals in various out-of-control situations, we show that the best conclusion from a signal from Rule A is that the process is in-control.

Thus, our conclusion is that we cannot validate the two above-mentioned arguments. Our analyses show consistently that only for small changes in the process variation it is possible to attain slightly better ARL values by using either Rule A or the *MR*-chart. However, this advantage is more than outweighed by considerably worse ARL values for detecting changes in the location of the process. For most practical purposes, adding the

MR-chart (or an equivalent) to the *X*-chart leads to a less powerful procedure that is likely to cause confusion. For this reason we advise against the use of the additional *MR*-chart or Rule A.

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Authors' biographies

Albert Trip graduated in 1983 in mathematics at the University of Groningen, the Netherlands. In the years thereafter he worked for Philips Electronics. As a statistical consultant he was involved in several projects at different plants where statistical data analysis and design of experiments was required. In the late 1980s he was recognized as the Taguchi-expert within Philips Electronics. In the 1990s he was more involved with introducing SPC (amongst others at Philips Semiconductors, Stadskanaal). His experiences with the introduction of SPC resulted in 1996 in a monograph, and in 2000 in a PhD from the University of Amsterdam. In 1997 he became a part-time consultant at the University of Amsterdam. Since 2001, most of his consulting has been for Sara Lee|DE. He has several publications in international journals and is co-author of several books on SPC. His research interests are control charts and the implementation and development of quality improvement programs.

Jaap Wieringa graduated in 1994 in econometrics at the University of Groningen, the Netherlands. He obtained his PhD in economics from the same university in 1999. Part of the research for his PhD thesis was carried out at Philips Semiconductors, Stadskanaal. From 1998 until 2001 he worked as a (senior) consultant at the University of Amsterdam. He was involved in implementing SPC and Six Sigma in several large companies including DAF Trucks (a Paccar Company), General Electric and Sara Lee|DE. He is currently working as an assistant professor in Marketing at the University of Groningen. He has several international publications both in the field of industrial statistics and in the field of marketing. He is co-author of a Dutch book on Six Sigma. His research interests are control charts, (multivariate) time series, econometric marketing model building, and market research.

